

Plus/Minus Grading

The Problem

In the Fall 2008 semester, the College of Liberal Arts and Sciences added grades with pluses and minuses (A-, B+, etc.) to its list of available grades. (Such grades had been available in some schools at KU previously, but not in the College.) Here are the web pages stating the KU and CLAS grade lists:

<http://www.registrar.ku.edu/~registr/transcript/overview.shtml>

<http://www.collegesas.ku.edu/policies/GPA.shtml>

When the only grades were A, B, C, D, and F, it was pretty easy to come up with a way of relating percentages to letter grades. For example, I usually used the following grading scale:

Table 1:

<u>letter grade</u>	<u>minimum percentage</u>
A	90
B	80
C	70
D	60
F	—

With the introduction of pluses and minuses, minimum percentages need to be determined for a much longer list of grades, as shown in table 2. (For now I'll assume the possibility of A+, even though it is not an available grade at KU, and then I'll adjust for its absence at the end.)

Table 2:

<u>letter grade</u>	<u>minimum percentage</u>
A+	?
A	?
A-	?
B+	?
B	?
B-	?
C+	?
C	?
C-	?
D+	?
D	?
D-	?
F	—

My Solution

If you think of the new grading scale as dividing each of the old grades' ranges into three smaller ranges (e.g., the A range gets divided into ranges for A+, A, and A-), it seems logical to think that the minimum percentage for an A- under my new grading scale should be the same as the minimum percentage for an A under my old grading scale, that the minimum percentage for a B- under my new grading scale should be the same as the minimum percentage for a B under my old grading scale, and so on. So I can partially fill in my table as follows:

Table 3:

<u>letter grade</u>	<u>minimum percentage</u>
A+	?
A	?
A-	90
B+	?
B	?
B-	80
C+	?
C	?
C-	70
D+	?
D	?
D-	60
F	—

Now, I'd like for the other minimum percentages to be based on KU's grade-point conversion scale:

Table 4:

<u>letter grade</u>	<u>grade points</u>
A+	—
A	4
A-	3.7
B+	3.3
B	3
B-	2.7
C+	2.3
C	2
C-	1.7
D+	1.3
D	1
D-	0.7
F	—

Notice, though, that there is a structural mismatch between tables 1–3 and table 4. The former were concerned with minimum values, and the latter is concerned with middle values (e.g., 3 grade points goes with the middle of the B range—it has nothing to do with the bottom of the B range). So, to use KU’s grade-point conversion scale to figure out my new percentage-based grading scale, I need to pay attention to middle percentages, not minimum percentages. My old grading scale had 95 in the middle of the A range, 85 in the middle of the B range, and so on. I’ll keep these as the middle percentages in my new grading scale:

Table 5:

<u>letter grade</u>	<u>middle percentage</u>
A+	—
A	95
A–	?
B+	?
B	85
B–	?
C+	?
C	75
C–	?
D+	?
D	65
D–	?
F	—

Now, to fill in the other percentages using KU’s grade-point conversion scale, I’ll assume that the relative spacing of the grades on KU’s grade-point conversion scale should dictate the relative spacing of the grades’ middle percentages on my grading scale. I’ll start with the B+. Since a B+ is worth 3.3 grade points, and 3.3 is 30 percent of the way from a B (3.0) to an A (4.0), I want to know what number is 30 percent of the way from an 85 to a 95. That number, of course, is 88. So, that should be the middle percentage for a B+. By similar reasoning (working with 3.7 grade points for an A–), the middle percentage for an A– should be a 92. Filling in the other values analogously, I have the following:

Table 6:

<u>letter grade</u>	<u>middle percentage</u>
A+	98
A	95
A–	92
B+	88
B	85
B–	82
C+	78
C	75
C–	72
D+	68
D	65
D–	62
F	—

With these middle values determined, all I need to do to figure out the corresponding ranges is to figure out the midpoints that lie between consecutive numbers:

Table 7:

<u>letter grade</u>	<u>middle percentage</u>	<u>midpoint (threshold)</u>
A+	98	
A	95	96.5
A–	92	93.5
B+	88	90
B	85	86.5
B–	82	83.5
C+	78	80
C	75	76.5
C–	72	73.5
D+	68	70
D	65	66.5
D–	62	63.5
F	—	—

Now, there are just a couple of further decisions I have to make. First, what should the threshold between D– and F be? For consistency with my old grading scale and for consistency between D– and the

other minus grades under my new grading scale, I'll make it 60.

Second, should a student whose percentage is equal to a threshold percentage get the letter grade just above it, or just below? For reasons of charity, I prefer to award the letter grade just above it. So, these thresholds are actually the minimum percentages for the grades just above them. That means that I can completely fill in the "minimum percentage" table I started with, but couldn't get very far with at the time (table 3):

Table 8:

<u>letter grade</u>	<u>minimum percentage</u>
A+	96.5
A	93.5
A-	90
B+	86.5
B	83.5
B-	80
C+	76.5
C	73.5
C-	70
D+	66.5
D	63.5
D-	60
F	—

So that's the way I would assign letter grades, under a complete plus/minus system. KU's grading system, however, maxes out at A; there is no A+. So I need to make an adjustment.

One way of handling this is to enlarge the intervals associated with each of the eleven remaining passing grades (A down to D-). But that would violate a lot of the constraints I've operated with so far, as well as being a mess.

A second option is to proportionally enlarge the intervals associated with A and A-, so that 90 remains the minimum value for A-, with 95 being the new minimum value for A. This, however, would make the unavailability of the A+ grade result in a disproportionately high percentage required in order to get an A. I think that would make A's harder to get than they should be.

A third option, my preferred one, is just to absorb the values associated with A+ into the range for A. So, the minimum value for an A would remain 93.5, and anything above that (up to 100, or higher, for that matter) would still be an A:

Table 9:

<u>letter grade</u>	<u>minimum percentage</u>
A	93.5
A-	90
B+	86.5
B	83.5
B-	80
C+	76.5
C	73.5
C-	70
D+	66.5
D	63.5
D-	60
F	—

So those are the minimum percentages I use. In most courses, I grade individual assignments on a scale of 0 to 100, and then at the end of the semester I use Blackboard or Excel to compute each student's final average to the nearest hundredth of a percentage point. So, in practice, my grading scale is as follows:

Table 10:

<u>letter grade</u>	<u>percentage range</u>
A	93.50 and above
A-	90.00–93.49
B+	86.50–89.99
B	83.50–86.49
B-	80.00–83.49
C+	76.50–79.99
C	73.50–76.49
C-	70.00–73.49
D+	66.50–69.99
D	63.50–66.49
D-	60.00–63.49
F	59.99 and below

Frequently Asked Questions

1. *The ranges for the plus/minus grades (such as B+ and B-) are 3.5 percentage points wide, but the ranges for the flat grades (such as B) are only 3 percentage points wide. Isn't that weird?*

Yes, considered by itself. But it reflects the fact that the grade points aren't themselves evenly spaced: there's a difference of 0.3 between some pairs of consecutive grade points (e.g., 3.0 and 3.3), but a difference of 0.4 between some others (e.g., 3.3 and 3.7). If the grade points were more evenly spaced (e.g., 3.00, 3.33, 3.67, etc.), then the mathematical technique used above (the one used to fill in

table 6) would yield more equally sized percentage-point ranges for the letter grades.

2. *Does that mean there's something fishy about the fact that pluses and minuses are worth only 0.3 instead of 0.33?*

No—the grades available in a grading system don't need to be equally spaced along whatever numerical scales (e.g., grade points or percentages) they can be correlated with.

3. *So why not just say that a B is anything between 83.33 and 86.67? Those numbers seem more intuitive, as thresholds, than 83.5 and 86.5.*

Those numbers do seem more intuitive, but they can have some counter-intuitive consequences. To see this, suppose that, partway through the semester, a student is right in the middle of the B+ range, with an average of 88.33 (which is halfway between 86.67 and 89.99). She works hard to raise her average by 3.33 percentage points, to 91.66, which is right in the middle of the A– range (it's halfway between 90.00 and 93.33). For her efforts, she is rewarded with an extra 0.4 grade points (3.7 grade points for an A– instead of 3.3 grade points for a B+). But now suppose this same student had been right in the middle of the B range, with an average of 85. Then her 3.3-percentage-point increase would have resulted in a final average of 88.33 (right in the middle of the B+ range, as we saw above). Then, her reward for her efforts would have been only 0.3 grade points (3.3 grade points for a B+ instead of 3 grade points for a B). It seems odd that the reward a student gets for raising her grade by a certain number of percentage points should depend, in this way, on where she started. My grading scale doesn't allow this to happen, since it makes the distance from the middle of the B+ range to the middle of the A– range larger than the distance from the middle of the B range to the middle of the B+ range.

Here's another way of looking at this issue. When thinking about pluses and minuses, it is natural to take $\frac{1}{3}$ as the default plus/minus increment: grade points would be raised and lowered by this amount, we would use percentages like $83\frac{1}{3}$ and $86\frac{2}{3}$, and so on. When this default is deviated from, as with our plus/minus increment of 0.3, the percentages need to be adjusted accordingly. Under the default system, a student with an 85—in the middle of the B range—would need to raise her average by $1\frac{2}{3}$

percentage points in order to get a B+. Her reward would be an extra $\frac{1}{3}$ grade points. If her reward is going to be less than that, then the extra work she has to do should be less, too. Specifically, if her reward is going to be only 0.3 grade points, then the extra percentage points she needs should be $1\frac{2}{3}$ times the ratio of 0.3 to $\frac{1}{3}$. That ratio is 0.9, and $1\frac{2}{3}$ times 0.9 is $1\frac{1}{2}$. Hence my B+ threshold of $86\frac{1}{2}$ rather than $86\frac{2}{3}$.