Ben Eggleston
University of Kansas, Fall 2021
Philosophy 666: Rational Choice Theory
December 3, 2021

name ______________________________

test 3 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions.

Questions:

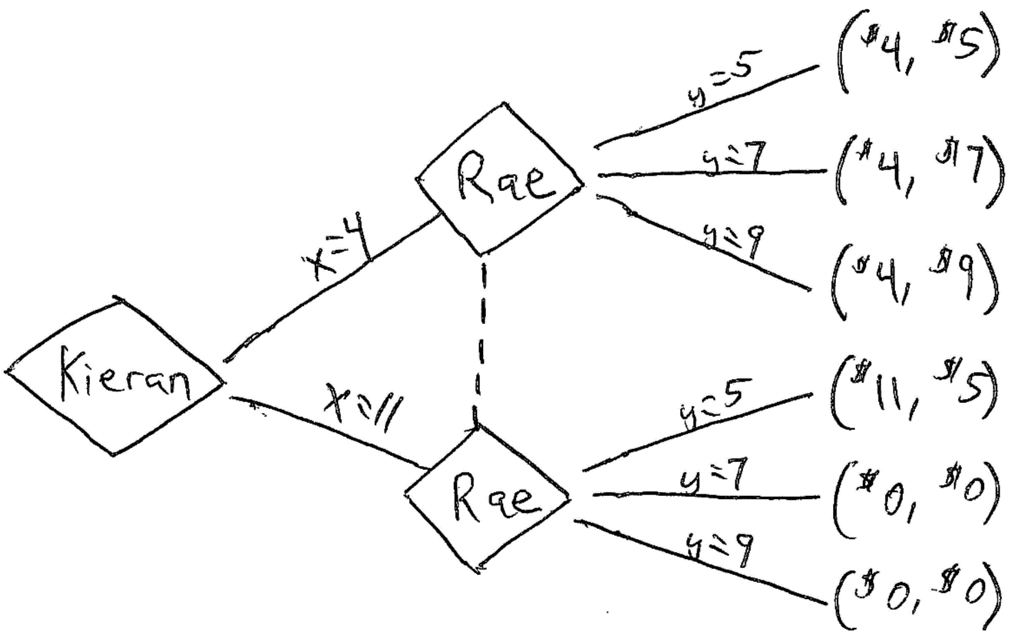
For questions 1 and 2, assume the following:

> Kieran can choose either $x = 4$ or $x = 11$. Then Rae, not knowing Kieran's choice, can choose $y = 5$ or $y = 7$ or $y = 9$. If $x + y > 17$, each player gets \$0; otherwise, Kieran gets \$$x$ and Rae gets \$$y$.

Also, when answering questions 1 and 2, when stating the outcomes, you do not have to write the players' names – you can just write ordered pairs of payoffs where the first payoff is Kieran's and the second is Rae's.

1. Draw a tree for this game.

   answer:



2. Draw a matrix (or matrices, if necessary) for this game.

answer:

			Rae	
		$y = 5$	$y = 7$	$y = 9$
Kieran	$x = 4$	(\$4, \$5)	(\$4, \$7)	(\$4, \$9)
	$x = 11$	(\$11, \$5)	(\$0, \$0)	(\$0, \$0)

Special instruction:

Below, matrices containing only one number in each cell represent zero-sum games in which the number given represents the row player's payoff and the column player's payoff is the negation of that.

3. Analyze the following game using dominance considerations and state the strategy pair that results from that analysis.

	C_1	C_2	C_3
R_1	6	3	4
R_2	7	4	5

answer:

Eliminate R_1 and C_1 and C_3:

	C_1	C_2	C_3
R_1	6	3	4
R_2	7	4	5

The remaining strategy pair is (R_2, C_2).

4. Analyze the following game using dominance considerations and state the strategy pair that results from that analysis.

	C_1	C_2	C_3	C_4
R_1	5	4	5	4
R_2	5	6	2	9
R_3	7	6	7	8

answer:

First, eliminate R_1 and C_1 and C_4:

	C_1	C_2	C_3	C_4
R_1	5	4	5	4
R_2	5	6	2	9
R_3	7	6	7	8

Second, eliminate R_2:

	C_1	C_2	C_3	C_4
R_1	5	4	5	4
R_2	5	6	2	9
R_3	7	6	7	8

Third, eliminate C_3:

	C_1	C_2	C_3	C_4
R_1	5	4	5	4
R_2	5	6	2	9
R_3	7	6	7	8

The remaining strategy pair is (R_3, C_2).

5. State whether the following game has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3
R_1	5	6	7
R_2	6	5	4

answer: no equilibrium strategy pairs consisting of pure strategies

6. Imagine a two-person zero-sum game in which both players have two pure strategies – R_1 and R_2 for the row player, and C_1 and C_2 for the column player. Suppose that when the row player plays a mixed strategy of the form (p R_1, ($1 - p$) R_2) against a mixed strategy for the column player of the form (q C_1, ($1 - q$) C_2), the row player's expected utility is $[p \times (15q - 7)] + (8q - 3)$. What mixed strategy should the column player play, if she would like to play a mixed strategy that could (along with a correctly chosen mixed strategy for the row player) be part of an equilibrium strategy pair? (Be sure to write a mixed strategy for the column player, not just the value of a variable.)

answer:

The column player should play the mixed strategy corresponding to the value of q that makes the row player's expected utility independent of the value of p. Given that the expression for the row player's expected utility has p multiplied by $15q - 7$, this latter expression should be set equal to 0. That yields the equation $15q - 7 = 0$, or $15q = 7$, or $q = 7/15$. So, the column player should play the mixed strategy ($7/15$ C_1, $8/15$ C_2).

7. What values of p and q make (p R_1, ($1 - p$) R_2; q C_1, ($1 - q$) C_2) an equilibrium strategy pair for the following game? (You do not have to show your work. An answer of the form 'p = __, q = __' can earn full credit.)

	C_1	C_2
R_1	8	5
R_2	3	7

answer:

$$p = \frac{4}{7}$$

$$q = \frac{2}{7}$$

(These can be computed as follows.)

$$p = \frac{7-3}{(7-3)+(8-5)} = \frac{4}{4+3} = \frac{4}{7}$$

$$q = \frac{7-5}{(7-5)+(8-3)} = \frac{2}{2+5} = \frac{2}{7}$$

8. Suppose that, for some game, the row player derives the expected utility, for him, of playing the strategy (p R_1, ($1 - p$) R_2), on the assumption that the column player is playing a certain strategy (do not worry about the details of the column player's strategy). Suppose the expected utility that the row player derives is $-3p + 7$. What is the row player's expected-utility-maximizing choice of p? What strategy does this mean that the row player should play?

answer:

Because the formula for the expected utility has a negative coefficient for p, the expected utility is maximized when p is minimized. Since the possible values of p range from 0 to 1, the minimum possible value for p is 0. That corresponds to the row player's strategy (0 R_1, ($1 - 0$) R_2), which is (0 R_1, 1 R_2), which is simply the pure strategy R_2.

9. Write the following matrix on one of your answer sheets and circle any equilibrium outcome(s). Also state whether this game is a coordination game, a prisoner's dilemma, or neither.

	C_1	C_2
R_1	17, 8	12, 9
R_2	19, 6	15, 7

answer:

	C_1	C_2
R_1	17, 8	12, 9
R_2	19, 6	[✓ 15, 7]

This game is a prisoner's dilemma.

10. Write the following matrix on one of your answer sheets and circle any equilibrium outcome(s). Also state whether this game is a coordination game, a prisoner's dilemma, or neither.

	C_1	C_2
R_1	18, 5	13, 4
R_2	11, 2	16, 7

answer:

	C_1	C_2
R_1	[✓ 18, 5]	13, 4
R_2	11, 2	[✓ 16, 7]

This game is a coordination game.

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.