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Philosophy 666: Rational Choice Theory
November 1, 2021

test 2 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Questions:

1. Suppose Lucia is shopping for a birthday present for her friend. She has found a reasonably good present, and is deciding whether to spend another hour continuing to shop, in order to find an excellent present. Her preferences over the possible outcomes can be represented with the following utility function:

item	utility
give excellent present	10
give reasonably good present	2
spend an additional hour shopping	–1

Suppose she believes that if she spends another hour shopping, she has a 40-percent chance of finding an excellent present (and will buy that rather than the reasonably good present). Even if she succeeds, though, she will experience the spent hour as a cost incurred, as indicated in the table above. Does the principle of maximizing expected utility counsel her to keep shopping in the hope of giving an excellent present, or stop shopping and give the reasonably good present?

answer:

$$EU(\text{keep shopping}) = \left(\frac{4}{10}\right)(10-1) + \left(1-\frac{4}{10}\right)(2-1)$$

$$= \left(\frac{4}{10}\right)(9) + \left(\frac{6}{10}\right)(1)$$

$$= \frac{36}{10} + \frac{6}{10}$$

$$= \frac{42}{10}$$

$$= 4.2$$

$$EU(\text{stop shopping and give reasonably good present}) = 2$$

So, the expected utility of continuing to shop is greater than the expected utility of stopping. So, the principle of maximizing expected utility would counsel her to keep shopping.

2. Suppose a potential arsonist has preferences that can be represented (in part) with the following utility function:

item	utility
committing arson	1.5
not committing arson, spending 0 years in prison	0
spending 0.25 years in prison	−0.38
spending 0.5 years in prison	−0.62
spending 0.75 years in prison	−0.82
spending 1 year in prison	−1.0
spending 2 years in prison	−1.6

Suppose he believes the following: (A) If he commits arson, the probability of punishment is 1/4. (B) If he does not commit arson, he will not be punished. (C) If he commits arson and gets punished, he will still regard the arson itself as a benefit (the enjoyment is not eliminated by the subsequent punishment, if it occurs). How severe must the punishment be, in order for the expected utility of committing arson to be lower than the expected utility of not committing arson? Write your answer as an equation or inequality of one of the following three forms:

u(punishment) = __ or u(punishment) > __ or u(punishment) < __

Of course, on the right side of your answer, instead of a blank ('__'), you will have a number or a numerical expression. That expression does not have to be simplified. For example, it could be some numbers (possibly integers, but not necessarily) that are added, subtracted, multiplied, and/or divided by each other. But the left side of your answer must be just 'u(punishment)', and the middle must be just '=' or '>' or '<'.

answer:

$$EU(\text{arson}) < EU(\text{no arson})$$

$$\left(\frac{1}{4}\right)(1.5 + u(\text{punishment})) + \left(1 - \frac{1}{4}\right)(1.5) < 0$$

$$\left(\frac{1}{4}\right)(1.5 + u(\text{punishment})) + \left(\frac{3}{4}\right)(1.5) < 0$$

$$1.5 + u(\text{punishment}) + (3)(1.5) < 0$$

$$1.5 + u(\text{punishment}) + 4.5 < 0$$

$$u(\text{punishment}) + 6 < 0$$

$$u(\text{punishment}) < -6$$

3. Suppose Jane is overseeing the construction of a tall building on a narrow lot surrounded by other buildings. The next phase of construction, as currently planned, has a 10-percent chance of damaging an adjacent building. Jane has the option of spending an extra $30,000 to use more specialized construction equipment that will eliminate the chance of damaging the adjacent building. How expensive must the potential damage to the adjacent building be, in order for the Hand rule to require Jane to incur that extra expense?

answer:

$$B < PL$$

$$\$30{,}000 < \left(\frac{1}{10}\right) L$$

$$\$300{,}000 < L$$

$$L > \$300{,}000$$

So, the potential damage would have to exceed \$300,000 in value, in order for the Hand rule to require Jane to spend the extra \$30,000 to do the job more carefully.

For questions 4 and 5, let L be a lottery that provides a 1/3 chance of winning a basket and a 2/3 chance of winning nothing. Also, assume that a plate is a separate possible prize. And assume the following:

u(plate) = u(nothing) + x

u(basket) = u(plate) + y

$x > 0$

$y > 0$

4. Suppose Chea prefers L to the plate. What constraint concerning x and y (in addition to the constraints just stated) implies utility assignments for the three prizes (nothing, plate, and basket) that make the principle of maximizing expected utility agree with Chea's preference? Show your work.

   answer:

$$\text{EU(L)} > \text{EU(plate)}$$

$$\left(\frac{1}{3}\right) u(\text{basket}) + \left(\frac{2}{3}\right) u(\text{nothing}) > u(\text{plate})$$

$$\left(\frac{1}{3}\right) [u(\text{nothing}) + x + y] + \left(\frac{2}{3}\right) u(\text{nothing}) > u(\text{nothing}) + x$$

$$u(\text{nothing}) + x + y + 2u(\text{nothing}) > 3u(\text{nothing}) + 3x$$

$$3u(\text{nothing}) + x + y > 3u(\text{nothing}) + 3x$$

$$x + y > 3x$$

$$y > 2x$$

5. What are some utility assignments for the three prizes that make the principle of maximizing expected utility agree with Chea's preference? Write your answer as a series of three equations, like this:

   u(nothing) = __

   u(plate) = __

   u(basket) = __

   (Of course, in each equation, instead of a blank ('__'), you will have a number.)

answer: the following, or many other possibilities:

u(nothing) = 0

u(plate) = 1

u(basket) = 10

6. Suppose Agathon prefers \$5 to a 1-in-a-million chance at a prize of \$1 million. Can we infer that Agathon is risk-averse? Why or why not?

   answer:

   No, we cannot. We can infer that a person is risk-averse only on the basis of a preference that cannot be explained simply by comparing the options' expected monetary values. Since Agathon's dispreferred option has an expected monetary value of \$1, and that is less than the \$5 that Agathon prefers, Agathon's preference can be explained simply by comparing the options' expected monetary values, and there is no basis for attributing risk aversion to Agathon.

7. Attempting to make the principle of maximizing expected utility compatible with the preferences of the chooser in the Allais paradox leads to the derivation of some constraints on some variables. Which of the following is the best description of those constraints?

   (a) There is actually only one constraint, but it is impossible to satisfy, as $x < x$ is impossible to satisfy.

   (b) There are two constraints, and each of them (taken individually) is impossible to satisfy, as $x < x$ and $y > y$ are each impossible to satisfy.

   (c) There are two constraints, and they are directly opposed to each other, similar to $x < y$ and $x > y$.

   (d) There are two constraints, and they specify a range of possible values for x and y, similar to $y > 2x$ and $y < 10x$.

   answer: c

8. Let L be a lottery comprising the following:

   - a 10-percent chance of winning macaroni
   - a 20-percent chance of winning spaghetti
   - a 70-percent chance at a lottery providing a 60-percent chance of winning macaroni and a 40-percent chance of winning spaghetti

   What non-compound lottery does L reduce to? Show your work.

   answer:

   possible prizes: macaroni, spaghetti

   $$\text{macaroni probability} = \left(\frac{1}{10}\right) + \left(\frac{7}{10}\right)\left(\frac{6}{10}\right) = \frac{10}{100} + \frac{42}{100} = \frac{52}{100}$$

   $$\text{spaghetti probability} = \left(\frac{2}{10}\right) + \left(\frac{7}{10}\right)\left(\frac{4}{10}\right) = \frac{20}{100} + \frac{28}{100} = \frac{48}{100}$$

   So, L reduces to the non-compound lottery providing a 52-percent chance of winning macaroni and a 48-percent chance of winning spaghetti, which can be written as L(52/100, macaroni, spaghetti).

9. Suppose Bastian's most-preferred prize is an airplane and his least-preferred prize is a tricycle. Suppose a car is an intermediate prize. What kind of preference does the continuity condition imply that Bastian holds?

   answer:

   It implies that he holds a preference of the form 'car I L(*p*, airplane, tricycle)', where *p* is some probability between 0 and 1.

10. What is the role of the better-odds condition in the proof of the representation theorem?

    answer:

    It facilitates the prediction of the chooser's preference regarding any two options by providing a prediction of the chooser's preference regarding two lotteries that are derived from the two options by other rationality conditions. For example, those other conditions might say option A is equivalent to the lottery L(0.7, best, worst), and they might say option B is equivalent to the lottery (0.4, best, worst), and the better-odds condition predicts that the chooser will prefer the former lottery to the latter.

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.