Ben Eggleston
University of Kansas, Spring 2017
Philosophy 666: Rational Choice Theory
April 28, 2017

test 3 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Questions:

1. The following is a zero-sum game written using condensed notation. State whether it has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form (R_x, C_y), where *x* and *y* are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	8	3	2	7
R_2	2	3	2	9
R_3	4	4	7	4

   answer: (R_3, C_2)

2. Draw the following matrix on one of your answer sheets and fill in each of the four cells with an integer between 1 and 9 (you can use up to four different integers, or you can repeat some integers) so that R_1 dominates R_2 and the outcome at (R_1, C_2) is the only equilibrium outcome.

	C_1	C_2
R_1		
R_2		

   answer: the following, or many other possibilities:

	C_1	C_2
R_1	6	5
R_2	2	1

3. In the following game, what is the expected utility, for the row player, of playing the strategy (p R_1, (1 – p) R_2), on the assumption that the column player is playing the strategy (1/4 C_1, 3/4 C_2)? Your answer should be of the form $xp + y$, where x and y are real numbers.

	C_1	C_2
R_1	8	2
R_2	3	6

answer:

EU(p R_1, (1 – p) R_2)

= $p \times \text{EU}(R_1) + (1 - p) \times \text{EU}(R_2)$

= $p \times [(1/4 \times 8) + (3/4 \times 2)] + (1 - p) \times [(1/4 \times 3) + (3/4 \times 6)]$

= $p \times (8/4 + 6/4) + (1 - p) \times (3/4 + 18/4)$

= $p \times 14/4 + (1 - p) \times 21/4$

= $p \times 14/4 + 21/4 + (-p \times 21/4)$

= $p \times 14/4 + 21/4 + p \times -21/4$

= $p \times -7/4 + 21/4$

= $-7p/4 + 21/4$

4. Suppose that, for some game, the row player derives the expected utility, for him, of playing the strategy (p R_1, (1 – p) R_2), on the assumption that the column player is playing a certain strategy (do not worry about the details of the column player's strategy). Suppose the expected utility that the row player derives is $-2p + 5$. What is the row player's utility-maximizing choice of p? What strategy does this mean that the row player should play?

   answer:

   The row player's utility-maximizing choice of p is $p = 0$; this means that the row player should play the strategy (0 R_1, (1 – 0) R_2), which is (0 R_1, 1 R_2), which is simply the pure strategy R_2.

5. Suppose that, for some game, the row player derives the expected utility, for him, of playing the strategy (p R_1, (1 – p) R_2), on the assumption that the column player is playing a certain strategy (do not worry about the details of the column player's strategy). Suppose the expected utility that the row player derives is 5. What does this mean about the effect of the row player's choice of p on the expected utility, for him, of playing the game, on the given assumption about the column player's strategy? What does this tell us about the strategy that it is assumed that the column player is playing?

   answer:

   It means that the row player's choice of p has no effect on the expected utility, for him, of playing the game. This tells us that the strategy that it is assumed that the column player is playing is eligible to be half of an equilibrium strategy pair for the game.

6. Derive the value of p that makes (p R_1, $(1 - p)$ R_2) eligible to be half of an equilibrium strategy pair for the following game. Derive this value of p by deriving an appropriate expression of the form q__ + __, where both blanks are filled in with the expressions of the form $xp + y$, where x and y are real numbers. Then proceed to derive the appropriate value of p.

	C_1	C_2
R_1	2	9
R_2	7	4

answer:

First, derive the expected utility, for the column player, of playing the strategy (q C_1, $(1 - q)$ C_2):

$$\mathrm{EU}(q\ C_1, (1 - q)\ C_2)$$

$$= q \times \mathrm{EU}(C_1) + (1 - q) \times \mathrm{EU}(C_2)$$

$$= q \times [(p \times -2) + (1 - p) \times -7)] + (1 - q) \times [(p \times -9) + (1 - p) \times -4)]$$

$$= q \times (-2p - 7 + 7p) + (1 - q) \times (-9p - 4 + 4p)$$

$$= q \times (5p - 7) + (1 - q) \times (-5p - 4)$$

$$= q \times (5p - 7) + 1 \times (-5p - 4) + (-q)(-5p - 4)$$

$$= q \times (5p - 7) + (-5p - 4) + q \times (5p + 4)$$

$$= q \times (5p - 7 + 5p + 4) + (-5p - 4)$$

$$= q \times (10p - 3) + (-5p - 4)$$

Then find the value of p that makes the value of the foregoing expression not depend on the column player's choice of q, by making its coefficient equal to 0:

$$10p - 3 = 0$$

$$10p = 3$$

$$p = 3/10$$

7. Write a 2 × 2 matrix that is an example of a prisoner's dilemma. Circle the equilibrium outcome(s).

answer: the following, or many other possibilities:

	C_1	C_2
R_1	2nd, 2nd	4th, 1st
R_2	1st, 4th	[✓ 3rd, 3rd]

8. Write a social preference ordering for Profile 2 (below) such that, if that social preference ordering and the social preference ordering given for Profile 1 were generated by some social welfare function called 'F', these two profiles and their corresponding social preference orderings would prove that F violates the independence of irrelevant alternatives condition. Be sure to write your answer on your answer sheets, regardless of whether you write it here.

Profile 1:

A	B	C	s.p.o.
a	*c*	*a*	*a*
b	*b*	*b*	*b*
c	*a*	*c*	*c*

Profile 2:

A	B	C	s.p.o.
a	*c*	*c*	
b	*b*	*a*	
c	*a*	*b*	

answer:

Any social preference ordering in which *a* is not ranked higher than *b* is a valid answer (since the relative standing of *a* and *b* does not change, for any individual, between Profile 1 and Profile 2). For example, the following:

b
a
c

9. What is an example of a profile that shows that pairwise majority rule violates condition U?

answer: the following, or many other possibilities:

A	B	C
a	*c*	*b*
b	*a*	*c*
c	*b*	*a*

10. What does the monotonicity condition require?

answer:

It requires that if an alternative gains standing in the preference ordering of any individual and does not lose standing in the preference ordering of any individual, then that alternative does not lose standing in the social preference ordering.

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.