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Philosophy 666: Rational Choice Theory
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# test 1 – answer key

***Instructions:***

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

***Questions:***

1. Suppose you hear a debate between a proponent of the completeness condition and an opponent of that condition – a debate involving a hypothetical example in which a person named Grant does not hold *a* P *b*, and does not hold *b* P *a*, and does not hold *a* I *b* – and afterwards you believe you heard a standard use of the small-improvement argument. Which of the following would be the best evidence for your belief?

   (a) The proponent said, "Okay, let us imagine a third option, *c*, such that *a* I *c* and *b* I *c*."

   (b) The opponent said the sentence mentioned in answer a.

   (c) The proponent said, "Okay, let us imagine a third option, *c*, that is slightly better than *a*."

   (d) The opponent said the sentence mentioned in answer c.

   *answer:* d

2. Suppose Shonda has the following six preferences:

   *a* P *b*        *a* P *c*        *a* P *d*        *b* P *c*        *b* P *d*        *c* P *d*

   Also, *s*uppose Shonda possess item *a*, but none of the others. If Marci wants to use Shonda as a money pump, which of the following offers should Marci make to Shonda?

   (a) "I see you have item *a*. If you give me that and 5 cents, I will give you item *b*."

   (b) "I see you have item *a*. If you give me that and 5 cents, I will give you item *c*."

   (c) "I see you have item *a*. If you give me that and 5 cents, I will give you item *d*."

   (d) none of the above – There is no reason to think Shonda can be used as a money pump.

   *answer:* d

3. Suppose Ray has these preferences: *a* P *b*, *b* P *c*, *c* P *d*. And suppose Ray's preference for *a* over *b* is one quarter as strong as his preference for *b* over *c*, which in turn is three times as strong as his preference for *c* over *d*. What is an interval utility function that accurately represents Ray's preferences?

   *answer:* the following, or any positive linear transformation of it:

| $x$ | $u(x)$ |
|---|---|
| $a$ | 19 |
| $b$ | 16 |
| $c$ | 4 |
| $d$ | 0 |

4. Suppose Marjorie has to take a trip, and she has to choose between driving and flying. If she drives and there is an accident, then her utility will be –60, and if she drives and there is no accident, then her utility will be 8. If she flies and there is an accident (the probability of which, let us say, is independent of the probability of a car accident), then her utility will be –900, and if she flies and there is no accident, then her utility will be 20. Set up a matrix for this situation so that the states of the world are mutually exclusive and exhaustive, and fill in the matrix.

   *answer:*

| | no car accident and no plane accident | no car accident, but plane accident | car accident, but no plane accident | car accident and plane accident |
|---|---|---|---|---|
| drive | 8 | 8 | –60 | –60 |
| fly | 20 | –900 | 20 | –900 |

The following choice matrix is for questions 5–7.

| | $S_1$ | $S_2$ |
|---|---|---|
| $A_1$ | $x$ | 2 |
| $A_2$ | 5 | 7 |

5. Suppose Olive believes that in state $S_1$, option $A_1$ will have a utility of 11. (That is, in Olive's opinion, $x$ = 11.) And suppose Olive decides to choose between options $A_1$ and $A_2$ using the optimism-pessimism rule, with an optimism index of 1/3. Which option would the rule recommend? Show your work.

   *answer:*

$$\alpha\text{-index for } A_1 = \left(\frac{1}{3}\right)(max) + \left(1 - \frac{1}{3}\right)(min) = \left(\frac{1}{3}\right)(11) + \left(\frac{2}{3}\right)(2) = \frac{11}{3} + \frac{4}{3} = \frac{15}{3}$$

$$\alpha\text{-index for } A_2 = \left(\frac{1}{3}\right)(max) + \left(1 - \frac{1}{3}\right)(min) = \left(\frac{1}{3}\right)(7) + \left(\frac{2}{3}\right)(5) = \frac{7}{3} + \frac{10}{3} = \frac{17}{3}$$

   $A_2$ has the higher α-index, so it is the option that the optimism-pessimism rule would recommend.

6. Suppose Edward also believes $x$ = 11, and decides to choose between options $A_1$ and $A_2$ using the minimax regret rule. Which option would the rule recommend? Show your work.

*answer:*

regret matrix (with an additional column):

| | $S_1$ | $S_2$ | maximum regret for each option: |
|---|---|---|---|
| $A_1$ | 0 | 5 | (5) |
| $A_2$ | 6 | 0 | 6 |

The smallest maximum regret is circled. Because it is in the row for $A_1$, that is the option that the rule would recommend.

7. Suppose Irving decides to choose between options $A_1$ and $A_2$ using the rule of maximizing expected utility using the principle of insufficient reason. If Irving applies the rule correctly and finds that it recommends $A_1$, what must he believe about $x$? (That is, what inequality of the form $x >$ __ or $x <$ __ must Irving believe is true?)

*answer:*

$$EU(A_1) > EV(A_2)$$

$$\left(\frac{1}{2}\right)(x) + \left(\frac{1}{2}\right)(2) > \left(\frac{1}{2}\right)(5) + \left(\frac{1}{2}\right)(7)$$

$$x + 2 > 5 + 7$$

$$x + 2 > 12$$

$$x > 10$$

8. Suppose a potential arsonist has preferences that can be represented with the following utility function:

| item | utility |
|---|---|
| committing arson | 1.2 |
| not committing arson, spending 0 years in prison | 0 |
| spending 0.25 years in prison | –0.38 |
| spending 0.5 years in prison | –0.62 |
| spending 0.75 years in prison | –0.82 |
| spending 1 year in prison | –1.0 |
| spending 2 years in prison | –1.6 |
| spending 3 years in prison | –2.2 |
| spending 4 years in prison | –2.6 |
| spending 5 years in prison | –3.1 |
| spending 10 years in prison | –5.0 |
| spending 20 years in prison | –8.1 |
| spending 40 years in prison | –13 |

Suppose the punishment for arson is spending 20 years in prison and the probability of the implementation of that punishment, in any case of arson, is $p$. Also, assume that if the potential arsonist does not commit arson, he will not spend any time in prison. What must the potential arsonist believe about $p$ in order for him to judge that the expected utility of committing arson is lower than the expected utility of not committing arson? Write your answer as an equation or inequality of one of the following three forms:

$p =$ __ or $p >$ __ or $p <$ __

Of course, on the right side of your answer, instead of a blank ('__'), you will have a number or a numerical expression. That expression does not have to be simplified. For example, it could be some numbers (possibly integers, but not necessarily) that are added, subtracted, multiplied, and/or divided by each other. But the left side of your answer must be just '*p*', and the middle must be just '=' or '>' or '<'.

*answer:*

$$EU(\text{arson}) < EU(\text{no arson})$$

$$(p)(1.2 - 8.1) + (1 - p)(1.2) < u(\text{no arson})$$

$$1.2p - 8.1p + 1.2 - 1.2p < 0$$

$$-8.1p + 1.2 < 0$$

$$-8.1p < -1.2$$

$$p > \frac{1.2}{8.1}$$

$$(\text{or anything equivalent to } p > \frac{4}{27})$$

9. Consider a variation on Newcomb's problem in which the transparent box contains only $10 and the opaque box might contain either $0 or $100 (with the contents of the opaque box being determined in the same way as in the original version of Newcomb's problem). How reliable must the predictor be in order for EMV(opaque) to be greater than EMV(both)? Your answer should be of the form $p >$ __, where the blank is filled in with some numerical expression. That expression should be a fraction that has two-digit numbers in the numerator and denominator, though a fraction that has three-digit numbers in the numerator and denominator could also earn full credit.

$$EMV(\text{opaque}) > EMV(\text{both})$$

$$(p)(\$100) + (1 - p)(\$0) > (p)(\$10) + (1 - p)(\$110)$$

$$\$100p > \$10p + \$110 - \$110p$$

$$\$100p > -\$100p + \$110$$

$$\$200p > \$110$$

$$p > \frac{\$110}{\$200}$$

$$p > \frac{110}{200}$$

$$p > \frac{11}{20}$$

10. Suppose all of the following: (1) you are taking a test consisting of multiple-choice questions that have nine answer choices each, (2) answering a question correctly increases your raw score by 6 points, (3) answering a question incorrectly decreases your raw score by 2 points, and (4) not answering a question does not increase or decrease your raw score. On any given question, what is the minimum number of incorrect answer choices that you must correctly eliminate in order for guessing randomly among the remaining answer choices to have a higher expected value (in terms of your raw score on the test) than not answering that question?

*answer:*

Let $e$ be the number of incorrect answers that you correctly eliminate. Then we solve as follows:

$$EV(\text{guess}) > EV(\text{blank})$$

$$\left(\frac{1}{9-e}\right)(6) + \left(\frac{9-e-1}{9-e}\right)(-2) > 0$$

$$\left(\frac{1}{9-e}\right)(6) + \left(\frac{8-e}{9-e}\right)(-2) > 0$$

$$\frac{6}{9-e} + \frac{2e-16}{9-e} > 0$$

$$\frac{2e-10}{9-e} > 0$$

$$2e - 10 > 0$$

$$2e > 10$$

$$e > 5$$

The smallest integer satisfying this inequality is 6. So, that is the minimum number of incorrect answers to correctly eliminate.

***Instructions, revisited:***

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.