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Philosophy 666: Rational Choice Theory
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test 1 – answer key

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Questions:

1. Suppose you hear a debate between a proponent of the completeness condition and an opponent of that condition – a debate involving a hypothetical example in which a person named Helen does not hold *a* P *b*, and does not hold *b* P *a*, and does not hold *a* I *b* – and afterwards you believe you heard a standard use of the small-improvement argument. Which of the following would be the best evidence for your belief?

   (a) The opponent said, "O.k., let us imagine a third option, *c*, such that *a* I *c* and *b* I *c*."

   (b) The opponent said, "O.k., let us imagine a third option, *c*, that is slightly better than *a*."

   (c) The proponent said, "Helen is irrational, because her preferences violate the completeness condition."

   (d) The proponent said, "The example of Helen is biased in your favor because *b* is only slightly better than *a*."

   answer: b

2. Suppose Mitt has the following six preferences:

   a P *b* *b* P *c* *c* P *a* *d* P *a* *d* P *b* *d* P *c*

   Also, suppose Mitt possess item *a*, but none of the others. If Norm wants to use Mitt as a money pump, which of the following offers should Norm make to Mitt?

   (a) "I see you have item *a*. If you give me that and 5 cents, I will give you item *b*."

   (b) "I see you have item *a*. If you give me that and 5 cents, I will give you item *c*."

   (c) "I see you have item *a*. If you give me that and 5 cents, I will give you item *d*."

   (d) none of the above – There is no reason to think Mitt can be used as a money pump.

   answer: b

3. Suppose Paula has these preferences: *a* P *b*, *b* P *c*, *c* P *d*. And suppose Paula's preference for *a* over *b* is three times as strong as her preference for *b* over *c*, which in turn is one half as strong as her preference for *c* over *d*. What is an interval utility function that accurately represents Paula's preferences?

answer: the following, or any positive linear transformation of it:

x	$u(x)$
a	6
b	3
c	2
d	0

4. Suppose that, tomorrow afternoon, Andy can either wash his car or see a movie. If he washes his car, then the outcome will be good if it does not rain right away and bad if it does. If he sees a movie, then the outcome will be great if the movie is interesting and bad if the movie is uninteresting. If you were to set up a standard matrix for this situation, which of the following would be the heading for one of the columns in your matrix?

   (a) 'wash car'

   (b) 'wash car and it rains right away'

   (c) 'it rains right away'

   (d) 'it rains right away and the movie is interesting'

   answer: d

5. Consider the choice situation represented by the following matrix.

	S_1	S_2	S_3
A_1	4	7	4
A_2	2	7	7
A_3	2	2	8
A_4	5	5	5
A_5	3	5	7

   5a. Which option(s) would be recommended by the minimax regret rule?

   answer: A_5

   5b. Which option(s) would be recommended by the method of maximizing expected utility using the principle of insufficient reason?

   answer: A_2

6. Suppose a potential arsonist has preferences that can be represented with the following utility function:

item	utility
committing arson	1.2
not committing arson, spending 0 years in prison	0
spending 0.25 years in prison	–0.38
spending 0.5 years in prison	–0.62
spending 0.75 years in prison	–0.82
spending 1 year in prison	–1.0
spending 2 years in prison	–1.6
spending 3 years in prison	–2.2

spending 4 years in prison	–2.6
spending 5 years in prison	–3.1
spending 10 years in prison	–5.0
spending 20 years in prison	–8.1
spending 40 years in prison	–13

Suppose the punishment for arson is spending 10 years in prison and the probability of the implementation of that punishment, in any case of arson, is *p*. Also, assume that if the potential arsonist does not commit arson, he will not spend any time in prison. What must the potential arsonist believe about *p* in order for him to judge that the expected utility of committing arson is lower than the expected utility of not committing arson? Write your answer as an equation or inequality of one of the following three forms:

p = __ or *p* > __ or *p* < __

Of course, on the right side of your answer, instead of a blank ('__'), you will have a number or a numerical expression. That expression does not have to be simplified. For example, it could be some numbers (possibly integers, but not necessarily) that are added, subtracted, multiplied, and/or divided by each other. But the left side of your answer must be just '*p*', and the middle must be just '=' or '>' or '<'.

answer: p > 1.2/5.0 (or anything equivalent to *p* > 0.24)

7. Suppose all of the following: (1) you are taking a test consisting of multiple-choice questions that have ten answer choices each, (2) answering a question correctly increases your raw score by 7 points, (3) answering a question incorrectly decreases your raw score by 2 points, and (4) not answering a question does not increase or decrease your raw score. On any given question, what is the minimum number of incorrect answer choices that you must correctly eliminate in order for guessing randomly among the remaining answer choices to have a higher expected value (in terms of your raw score on the test) than not answering that question?

   answer:

   Let *e* be the number of incorrect answers that you correctly eliminate. Then we solve as follows:

$$EV(geuss) > EV(blank)$$

$$\left(\frac{1}{10-e}\right)(7) + \left(\frac{10-e-1}{10-e}\right)(-2) > 0$$

$$\left(\frac{1}{10-e}\right)(7) + \left(\frac{9-e}{10-e}\right)(-2) > 0$$

$$\frac{7}{10-e} + \frac{2e-18}{10-e} > 0$$

$$\frac{2e-11}{10-e} > 0$$

$$2e - 11 > 0$$

$$2e > 11$$

$$e > \frac{11}{2}$$

Since *e* must be an integer, we have

$$e \geq 6$$

So, the minimum number of incorrect answers to correctly eliminate is 6.

For questions 8 and 9, let L be a lottery that provides a 1/3 chance of winning \$900 and a 2/3 chance of winning \$0. Also, assume the following:

$u(\$200) = u(\$0) + x$

$u(\$900) = u(\$200) + y$

$x > 0$

$y > 0$

8. Suppose Yolanda prefers \$200 to L. What constraint concerning x and y (in addition to the constraints just stated) implies utility assignments for the three dollar amounts (\$0, \$200, and \$900) that make the principle of maximizing expected utility agree with Yolanda's preference?

   answer: $2x > y$

9. What are some utility assignments for the three dollar amounts (\$0, \$200, and \$900) that make the principle of maximizing expected utility agree with Yolanda's preference? Write your answer as a series of three equations, like this:

   $u(\$0) = \_\_$

   $u(\$200) = \_\_$

   $u(\$900) = \_\_$

   (Of course, in each equation, instead of a blank ('__'), you will have a number.)

   answer: the following, or many other possibilities:

   $u(\$0) = 0$

   $u(\$200) = 1$

   $u(\$900) = 2$

10. Attempting to make the principle of maximizing expected utility compatible with the preferences of the chooser in the Allais paradox leads to the derivation of some constraints on some variables. Which of the following is the best description of those constraints?

    (a) There is actually only one constraint, but it is impossible to satisfy, as $x < x$ is impossible to satisfy.

    (b) There are two constraints, and each of them (taken individually) is impossible to satisfy, as $x < x$ and $y > y$ are each impossible to satisfy.

    (c) There are two constraints, and they are directly opposed to each other, similar to $x < y$ and $x > y$.

    (d) There are two constraints, and they specify a range of possible values for x and y, similar to $y > 2x$ and $y < 10x$.

    answer: c

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.