

Test on Social Choice Theory – Answer Key (#1b, 2b, 5, 6, 9)

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Special instruction:

The following two profiles and corresponding social preference orderings are referred to in some of the questions given below. Assume that the social preference orderings were generated by some social welfare function called ‘F’.

<u>Profile 1:</u>				<u>Profile 2:</u>			
<u>A</u>	<u>B</u>	<u>C</u>	<u>s.p.o.</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>s.p.o.</u>
<i>a</i>	<i>c</i>	<i>d</i>	<i>a</i>	<i>a</i>	<i>c</i>	<i>a</i>	<i>c</i>
<i>d</i>	<i>b, d</i>	<i>a</i>	<i>c</i>	<i>d</i>	<i>b, d</i>	<i>d</i>	<i>a</i>
<i>b</i>	<i>a</i>	<i>b, c</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b, c</i>	<i>b</i>
<i>c</i>			<i>d</i>	<i>c</i>			<i>d</i>

Questions:

1. *Borda count:*
 - 1a. What is the social preference ordering that the Borda count would generate for profile 1?
 Next to each alternative in the ordering you wrote, write the score that the Borda count would give for it.
 - 1b. Does the Borda count satisfy the Pareto condition?
 (You can just write ‘Yes.’ or ‘No.’ You do not need to explain your answer.)

 Yes.
2. *Pairwise majority rule:*
 - 2a. What are the social preferences that pairwise majority rule would generate for profile 1?
 (You do not have to compile the social preferences into a social preference ordering.)
 - 2b. Does pairwise majority rule satisfy the independence of irrelevant alternatives condition?
 (You can just write ‘Yes.’ or ‘No.’ You do not need to explain your answer.)

 Yes.

3. *Pairwise majority rule, continued:*

What is an example of a profile that shows that pairwise majority rule violates condition U?

(Profiles 1 and 2 might be useless for this question; your best bet is probably to just write another profile.)

4. *Plurality voting and instant runoff voting:*

Let profile 3 be the following:

<u>A, B</u>	<u>C, D, E</u>	<u>F, G, H</u>	<u>I, J, K, L</u>
<i>c</i>	<i>a</i>	<i>b</i>	<i>d</i>
<i>a</i>	<i>c, d</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>b</i>	<i>a</i>	<i>a</i>
<i>d</i>		<i>c</i>	<i>b</i>

4a. What is the social preference ordering that plurality voting would generate for profile 3?

4b. What is the social preference ordering that instant runoff voting would generate for profile 3?

5. Which one of the following statements is true? If your answer is that statement a is true or that statement b is true, indicate a pair of alternatives that would be mentioned in an explanation of why it is true. (You do not have to provide the explanation.)

- Profiles 1 and 2 and their corresponding F-determined social preference orderings entail that F satisfies condition I.
- Profiles 1 and 2 and their corresponding F-determined social preference orderings entail that F violates condition I.
- Profiles 1 and 2 and their corresponding F-determined social preference orderings do not entail either that F satisfies condition I or that F violates condition I.

b; alternatives *a* and *c*

6. Which one of the following statements is true? If your answer is that statement a is true or that statement b is true, indicate an alternative that would be mentioned in an explanation of why it is true. (You do not have to provide the explanation.)

(same answer choices as for no. 5, but with the phrase ‘condition I’ replaced by the phrase ‘condition M’)

b; alternative *a*

7. Consider the following paragraph. Assume that TS, which the following paragraph refers to, is a true statement concerning some of the conditions on social welfare functions.

“For any social welfare function G that satisfies the conditions mentioned in the antecedent of statement TS, condition ___ allows us to assert the existence of a profile whose G-determined corresponding social preference ordering ranks alternative *a* above alternative *b*. Call that profile P1. Then, consider the profile that results if you start with P1, and you move *a* to the top of each person’s individual preference ordering. Call that profile P2. Condition ___ allows us to say that for P2, its G-determined social preference ordering ranks *a* above *b*. Finally, because *a* is ranked above *b* by every person in P2, condition ___ allows us to say that for *any* profile in which *a* is ranked above *b* by every person, its G-determined social preference ordering ranks *a* above *b*. This means that G satisfies condition ___.”

How should the four blanks be filled in? (Your answer should consist of a list of four words or abbreviations. Be sure the *order* of the four items on your list is unmistakable.)

8. What does Arrow’s impossibility theorem say?

9. Suppose set S is decisive for alternative k over alternative r . What do we need to know about some profile in order to legitimately infer that k is ranked above r in the social preference ordering corresponding to that profile?

We need to know that k is ranked above r in the individual preference orderings of all of the members of S .

10. Suppose the following: social welfare function H satisfies conditions P and L, person 1 has control (in the condition-L sense) over a versus b , and person 2 has control (in the condition-L sense) over c versus d . (As usual, assume that a , b , c , and d are distinct from each other, rather than there being any alternative that is referred to using multiple names.) Give an example of a profile that shows that H violates condition U.