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Philosophy 666: Rational Choice Theory
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Test on Game Theory – Answer Key (#1, 3, 7)

Instructions:

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period. No credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

Special instruction:

Most of the matrices given below have only one number in each cell. That number should be regarded as the row player's payoff, with the column player's payoff being the negation of that number.

Questions:

1. Suppose that a certain game is played by player 1 choosing $x = 1$, $x = 2$, or $x = 3$, then player 2 (with knowledge of player 1's choice) choosing $y = 4$ or $y = 5$, then payoffs to both players being some function of x and y (which is common knowledge between the players). Give an example of one of player 1's strategies, and state how many strategies player 1 has. Then give an example of one of player 2's strategies, and state how many strategies player 2 has.

   One of player 1's strategies is '$x = 1$', and player 1 has three strategies.

   One of player 2's strategies is 'If $x = 1$, then set $y = 4$; if $x = 2$, then set $y = 4$; if $x = 3$, then set $y = 4$', and player 2 has eight strategies.

2. Analyze the following game using dominance considerations and write the strategy pair(s) corresponding to its solution(s). Write each strategy pair in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	5	1	4	8
R_2	5	6	6	7
R_3	3	5	4	3

3. State whether the following game has any equilibrium strategy pair(s). (You can ignore mixed strategies and focus on pure strategies only.) If it does, write it (or each of them) in the form (R_x, C_y), where *x* and *y* are integers corresponding to row and column numbers, respectively.

	C_1	C_2	C_3	C_4
R_1	6	5	9	8
R_2	7	2	8	6
R_3	8	5	2	2

(R_1, C_2)

4. Derive the values of *p* and *q* that make (*p* R_1, (1 – *p*) R_2; *q* C_1, (1 – *q*) C_2) an equilibrium strategy pair for the following game. To derive each value, start with either the equation EU(R_1) = EU(R_2) or the equation EU(C_1) = EU(C_2) – whichever is appropriate – and show your work. Conclude each derivation with an equation of the form '*p* = __' or '*q* = __'.

	C_1	C_2
R_1	9	4
R_2	2	5

5. What values of *p* and *q* make (*p* R_1, (1 – *p*) R_2; *q* C_1, (1 – *q*) C_2) an equilibrium strategy pair for the following game? (You do not have to show your work. An answer of the form '*p* = __, *q* = __' can earn full credit.)

	C_1	C_2
R_1	5	7
R_2	6	3

6. Imagine a two-person zero-sum game in which both players have two pure strategies, and in which it is known to both players that the row player is playing the mixed strategy (1/2 R_1, 1/2 R_2). The column player realizes that if she also plays a mixed strategy giving equal probabilities to each of her pure strategies (that is, if she plays the mixed strategy (1/2 C_1, 1/2 C_2)), then her expected utility is as large as it would be with any other mixed strategy that she might choose to play (assuming, throughout, the mixed strategy (1/2 R_1, 1/2 R_2) for the row player). Does this mean that [(1/2 R_1, 1/2 R_2), (1/2 C_1, 1/2 C_2)] is an equilibrium strategy pair for this game? Why or why not?

7. Suppose that, in the following game, the row player knows that the column player is going to play the mixed strategy (2/3 C_1, 1/3 C_2). What is the expected utility (for the row player) of his general strategy (p R_1, (1 – p) R_2)? (Your answer to that question should be some expression involving the variable *p*.) Of all of the row player's pure and mixed strategies, which one has the greatest expected utility for him? (Your answer to that question should be some specific pure or mixed strategy, not an expression involving any variable.)

	C_1	C_2
R_1	8	4
R_2	5	7

EU(p R_1, (1 – p) R_2)
= $p \times$ EU(R_1) + (1 – p) $\times$ EU(R_2)
= $p \times [(2/3)(8) + (1/3)(4)] + (1 - p) \times [(2/3)(5) + (1/3)(7)]$
= $p \times (16/3 + 4/3) + (1 - p) \times (10/3 + 7/3)$
= $(p \times 20/3) + (1 - p) \times (17/3)$
= $20p/3 + 17/3 - 17p/3$
= $3p/3 + 17/3$
= $p + 17/3$

Since this is an increasing function of p, the row player maximizes his expected utility by setting $p = 1$. That means playing the pure strategy R_1.

8. Write a 2 × 2 matrix that is an example of a coordination game. Circle the equilibrium outcome(s).

9. Write a 2 × 2 matrix that is an example of a prisoner's dilemma. Circle the equilibrium outcome(s).

10. Is (or are) any of the outcomes of the following game Pareto optimal? If so, circle them. If not, write 'No.'

	C_1	C_2
R_1	8, 4	6, 8
R_2	4, 9	6, 6

Instructions, revisited:

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.