

## Test on Social Choice Theory

**Instructions:**

1. Answer all of the following questions on the answer sheets provided. You can write on this list of questions, but credit will be awarded only for answers written on answer sheets.
2. Do not access any book, notebook, newspaper, calculator, computer, cell phone, or other possible source of inappropriate aid during the test, do not leave the room before you are finished taking the test, and be sure to finish the test within this 50-minute testing period—no credit will be given for any work done after you access any possible source of inappropriate aid, after you leave the room for any reason, or after the end of the testing period.
3. When you are finished, be sure your name is written on each of your answer sheets, and turn them in. Also, turn in this list of questions. If you write your name on it, it will be returned with your graded answer sheets.

**Special instruction:**

The following two profiles and corresponding social preference orderings are referred to in some of the questions given below. Assume that the social preference orderings were generated by some social welfare function called ‘F’.

<u>Profile 1:</u>				<u>Profile 2:</u>			
<u>A</u>	<u>B</u>	<u>C</u>	<u>s.p.o.</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>s.p.o.</u>
<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>d</i>	<i>a</i>
<i>b</i>	<i>a, d</i>	<i>d</i>	<i>a</i>	<i>b</i>	<i>a, d</i>	<i>c</i>	<i>c</i>
<i>c</i>	<i>c</i>	<i>b</i>	<i>d</i>	<i>c</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>d</i>		<i>a</i>	<i>b</i>	<i>d</i>		<i>a</i>	<i>b</i>

**Questions:**

1. *Borda count:*
  - 1a. What is the social preference ordering that the Borda count would generate for profile 1?
  - 1b. What is a condition on social welfare functions that the Borda count violates?
2. *Pairwise majority rule:*
  - 2a. What is the social preference ordering that pairwise majority rule would generate for profile 2?
  - 2b. What is a condition on social welfare functions that pairwise majority rule violates?
3. *Pairwise majority rule, continued:*
  - 3a. Can either profile 1 or profile 2 be used to show that pairwise majority rule violates the condition you just mentioned (in your answer to question 2b)? Why or why not?
  - 3b. *This sub-question is optional. If you answered question 3a correctly, you have already earned full credit for question 3. But if you are concerned that you might not have answered question 3a correctly and would like*

*to guarantee yourself at least half credit for question 3, you can answer the following question: What is an example of a profile that shows that pairwise majority rule violates a condition on social welfare functions?*

4. *Instant runoff voting:*

4a. What is the social preference ordering that instant runoff voting would generate for the following profile?

<u>A, B</u>	<u>C, D, E</u>	<u>F, G, H</u>	<u>I, J, K, L</u>
<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>b</i>	<i>a, d</i>	<i>d</i>	<i>a</i>
<i>c</i>	<i>c</i>	<i>b</i>	<i>b</i>
<i>d</i>		<i>a</i>	<i>c</i>

4b. What is a condition on social welfare functions that instant runoff voting violates?

5. Which one of the following statements is true?

- Profiles 1 and 2 and their corresponding F-determined social preference orderings entail that F satisfies condition I.
- Profiles 1 and 2 and their corresponding F-determined social preference orderings entail that F violates condition I.
- Profiles 1 and 2 and their corresponding F-determined social preference orderings do not entail either that F satisfies condition I or that F violates condition I.

6. Same question as no. 5, but replace ‘condition I’ with ‘condition M’ in the three answer choices.

7. Can it be ascertained from profiles 1 and 2 and their corresponding F-determined social preference ordering whether F satisfies or violates condition NI? If so, how? If not, what additional information is needed?

8. Same question as no. 7, but replace ‘condition NI’ with ‘condition ND’.

9. What does Arrow’s impossibility theorem say?

10. Suppose that social welfare function G satisfies conditions P and L, person 1 has control (in the condition-L sense) over *a* versus *b*, and person 2 has control (in the condition-L sense) over *a* versus *c*. (As usual, assume that *a*, *b*, and *c* are distinct from each other, rather than there being any alternative that is referred to using multiple names.) Give an example of a profile that shows that G does not satisfy condition U.

***Instructions, revisited:***

As stated in item 3 of the instructions, turn in this list of questions along with your answer sheets.