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University of Kansas, Spring 2011
Philosophy 666: Rational Choice Theory

Questions for Credit

rules:

1. You can get help and advice from any source, as long as you fully comprehend the answers you turn in.
2. Answers are due in writing at the beginning of class unless otherwise specified.
3. If you are going to miss class, you can drop your answers off at my office or submit them by e-mail, but you must turn them in by the time class starts.
4. Whenever possible, please submit your answers in hard copy rather than by e-mail.

questions due Wednesday, January 26 (introduction and section 1.1):

1. What are the two main things rational choice theory is concerned with?

   (1) the rules and principles that you must comply with, in making choices, if your choices are to be rational and (2) the conditions that your preferences must satisfy, in order for them to be rational

2. Informally give an example of a choice situation that would most appropriately be handled by decision theory. (It does not have to be different from the situation you thought of for class on Monday, January 24.)

   (many correct answers)

3. Represent that choice situation using an outline of the kind discussed in section 1.1.

   (many correct answers)

questions due Friday, January 28 (sections 1.2–1.3):

4. Informally give an example of a choice situation that is a situation of choice under risk. (It does not have to be different from the situation you thought of for class on Wednesday, January 26.)

   (many correct answers)

5. Write a paragraph in which some preferences over at least four items are stated in ordinary conversational English. Then re-state that same information formally, using ‘P’, ‘I’, abbreviations or short names for the items, and/or a glossary, as needed.

   (many correct answers)

questions due Monday, January 31 (sections 1.4–1.5):

6. Suppose a person considers options {*j*, *k*, *l*} and forms the following preferences: *j* P *k*, *k* P *l*, *l* P *j*. Do these preferences satisfy the completeness condition?

   Yes, since there is exactly on preference stared for every pair of options. The fact that the preferences are cyclical prevents them from satisfying the transitivity condition, but not the completeness condition.

7. Suppose a person considers options {*m*, *n*, *o*, *p*}and forms the following preferences: *m* P *n*, *n* P *m*. If the person proceeds to specify preferences over all of the other pairs of options, could her preferences satisfy the completeness condition?

   No. The completeness conditions requires that the person hold *exactly one* of (1) *m* P *n*, (2) *n* P *m*, and (3) *m* I *n*. The person already violates the completeness condition by holding two of those three, and no further specifications of preferences can change that.

8. Consider the following preferences: *x* P *y*, *y* P *z*, *z* P *w*, *w* P *x*. Prove that these preferences violate the transitivity condition.

#	claim	justification
1	*x* P *y*	given
2	*y* P *z*	given
3	*z* P *w*	given
4	*w* P *x*	given
5	*x* P *z*	1 and 2, transitivity condition
6	*x* P *w*	5 and 3, transitivity condition
7	contradiction	6 and 4, completeness condition

9. Give an example in which a person has preferences that make him or her vulnerable to being used as a money pump.

   Any answer specifying cyclical preferences would be correct.

questions due Monday, February 7 (section 2.2):

Let preference ordering P_1 be the following:

e, *f*
b
a, *d*
c, *g*

10. Suppose I try to represent the preferences in P_1 with a utility function that includes the following utility assignments: $u(a) = 0$, $u(b) = 1$, and $u(c) = 2$. Could this be part of an ordinal utility function that correctly represents the preferences in P_1? Why or why not?

    No, it could not, because P_1 implies that *a* P *c*, and so it is necessary that $u(a) > u(c)$. But the values given do not comply with this requirement, since it is not the case that $0 > 2$.

11. Consider the preferences in P_1 and suppose that the preference for *e* over *b* is three times as strong as the preference for *b* over *a* and one fourth as strong as the preference for *f* over *g*. Provide an interval utility function that represents these preferences.

    There are infinitely many correct answers, but they are all positive linear transformations of this one:

x	*u*(*x*)
e, *f*	12
b	9
a, *d*	8
c, *g*	0

12. Provide a utility function that is a positive linear transformation of the one you provided in answer to question 11.

    Again, there are infinitely many correct answers, but they are all positive linear transformations of this one:

x	*u*(*x*)
e, *f*	121
b	91
a, *d*	81
c, *g*	1

questions due Wednesday, February 9 (sections 2.3–2.4):

13. Suppose you are working in an alchemy lab and are deciding whether to use proteins or radiation in your next procedure. Your leaves can be in state A, B, or C, and your roots can be in state P or Q. If you use proteins, you'll end up with neon if your leaves are in state A and your roots are in state P, argon if your leaves are in state C and your roots are in state P, and oxygen otherwise. If you use radiation, you'll end up with nitrogen if your leaves are in state A or your roots are in state Q, and krypton otherwise. Set up and fill in the matrix for this situation. Make sure that no two columns have identical outcomes in every row, even if this means combining what might originally appear to be states of the world that should be listed separately.

	leaves = A, roots = P	leaves = B, roots = P	leaves = C, roots = P	roots = Q
proteins	neon	oxygen	argon	oxygen
radiation	nitrogen	krypton	krypton	nitrogen

The following matrix is for questions 14–16.

	S_1	S_2	S_3
A_1	5	4	3
A_2	2	x	3

14. What inequality must x satisfy in order for option A_1 to be uniquely selected by the dominance principle?

    $x \leq 4$

15. What inequality must x satisfy in order for option A_1 to be uniquely selected by the maximin rule?

    Any x will work, for either the basic maximin rule or the lexical maximin rule.

16. What inequality must x satisfy in order for option A_1 to be uniquely selected by the maximax rule?

    $x < 5$ for the basic maximin rule, or $x \leq 5$ for the lexical maximin rule.

questions due Friday, February 11 (sections 2.5–2.6):

The following matrix is for questions 17–20.

	S_1	S_2	S_3	S_4
A_1	3	7	1	4
A_2	2	2	5	4
A_3	x	3	5	2

17. What must be true of x in order for option A_3 to be uniquely selected by the optimism-pessimism rule if $\alpha = 2/5$? (Hint: This involves computing the α-index of the other two options, and then figuring out what must be true of x in order for the α-index of A_3 to be the largest of the three α-indexes.)

α-index of A_1 = (2/5)(7) + (1 – 2/5)(1) = (2/5)(7) + (3/5)(1) = 14/5 + 3/5 = 17/5

α-index of A_2 = (2/5)(5) + (1 – 2/5)(2) = (2/5)(5) + (3/5)(2) = 10/5 + 6/5 = 16/5

The larger of 17/5 and 16/5 is, obviously, 17/5.

Now let's see what follows from assuming that the α-index of A_3 is greater than 17/5.

First, we will suppose that x is A_3's *minimum* possible value:

$(2/5)(5) + (1 - 2/5)(x) > 17/5$
$(2/5)(5) + (3/5)(x) > 17/5$
$10/5 + 3x/5 > 17/5$
$10 + 3x > 17$
$3x > 7$
$x > 7/3$

If $x > 7/3$, then it will not be A_3's minimum possible value (for that will be 2). So it will not work to suppose that x is A_3's minimum possible value; rather, we must suppose that x is A_3's *maximum* possible value:

$(2/5)(x) + (1 - 2/5)(2) > 17/5$
$(2/5)(x) + (3/5)(2) > 17/5$
$2x/5 + 6/5 > 17/5$
$2x + 6 > 17$
$2x > 11$
$x > 11/2$

If $x > 11/2$, then it will indeed be A_3's maximum possible value (since the maximum of its other possible values is 5), so this answer does not contradict the supposition that led to it (unlike when we supposed that x was A_3's minimum possible value). So, having derived $x > 11/2$ as above, we can be confident that it is correct.

18. What is the regret matrix that follows from this utility matrix, if $x > 3$?

	S_1	S_2	S_3	S_4
A_1	$x - 3$	0	4	0
A_2	$x - 2$	5	0	0
A_3	0	4	0	2

19. What must be true of x in order for option A_3 to be uniquely selected by the minimax regret rule?

$x > 7$

20. What must be true of x in order for option A_3 to be uniquely selected by the approach of maximizing expected utility using the principle of insufficient reason?

$x > 5$

questions due Wednesday, February 16 (section 3.2):

For questions 21–23, assume the following (and notice that these lotteries involve four different amounts of money):

L_1 is a lottery with a 2/3 chance at \$80 and a 1/3 chance at \$10.

L_2 is a lottery with a 2/3 chance at \$60 and a 1/3 chance at \$30.

L_3 is a lottery with a 1/2 chance at \$80 and a 1/2 chance at \$10.

$u(\$30) = u(\$10) + x$

$u(\$60) = u(\$30) + y = u(\$10) + x + y$

$u(\$80) = u(\$60) + z = u(\$10) + x + y + z$

21. Suppose more money is preferred to less, and that L_1 is preferred to L_2. What constraint concerning x, y, and z can be derived from these suppositions? (You can regard the constraints $x > 0$, $y > 0$, and $z > 0$ as assumed rather than stating them.)

$$\begin{aligned}
EU(L_1) &> EU(L_2) \\
(2/3)u(\$80) + (1/3)u(\$10) &> (2/3)u(\$60) + (1/3)u(\$30) \\
2u(\$80) + u(\$10) &> 2u(\$60) + u(\$30) \\
2[u(\$10) + x + y + z] + u(\$10) &> 2[u(\$10) + x + y] + u(\$10) + x \\
2u(\$10) + 2x + 2y + 2z + u(\$10) &> 2u(\$10) + 2x + 2y + u(\$10) + x \\
2x + 2y + 2z &> 2x + 2y + x \\
2x + 2z &> 3x \\
2z &> x
\end{aligned}$$

22. Suppose more money is preferred to less, and that L_2 is preferred to L_3. What constraint concerning x, y, and z can be derived from these suppositions? (Again, you can regard the constraints $x > 0$, $y > 0$, and $z > 0$ as assumed rather than stating them.)

$$\begin{aligned}
EU(L_2) &> EU(L_3) \\
(2/3)u(\$60) + (1/3)u(\$30) &> (1/2)u(\$80) + (1/2)u(\$10) \\
4u(\$60) + 2u(\$30) &> 3u(\$80) + 3u(\$10) \\
4[u(\$10) + x + y] + 2[u(\$10) + x] &> 3[u(\$10) + x + y + z] + 3u(\$10) \\
4u(\$10) + 4x + 4y + 2u(\$10) + 2x &> 3u(\$10) + 3x + 3y + 3z + 3u(\$10) \\
4x + 4y + 2x &> 3x + 3y + 3z \\
6x + 4y &> 3x + 3y + 3z \\
3x + y &> 3z
\end{aligned}$$

23. What are values of x, y, and z that satisfy both of the constraints you derived? What are utilities for the four amounts of money that fit these values of x, y, and z?

    There are infinitely many such values, but here is one set: $x = 1$, $y = 1$, $z = 1$.

    There are also infinitely many utilities for the four amounts of money they fit these values of x, y, and z (since there are infinitely many eligible values of $u(\$10)$, but if we just let $u(\$10) = 0$, here are the values that result from these values of x, y, and z: $u(\$10) = 0$, $u(\$30) = 1$, $u(\$60) = 2$, $u(\$80) = 3$.

questions due Friday, February 18 (section 3.3):

24. Suppose L_1 is a lottery that Philip regards as giving him a 1/4 chance at \$4,000. Suppose also that he prefers L_1 to \$8,000. Is he risk averse, risk neutral, or risk seeking, or do we not have enough information to say for sure?

    One answer earning credit is that he is risk seeking, since he prefers the risky option with a smaller expected monetary value (\$1,000) to the larger sum of money (\$8,000).

    But another answer earing credit is that we do not have enough information to say for sure, on the grounds that (1) we do not know the other component(s) of the lottery L_1 and (2) the information given suggests that Philip violates our usual assumption that more money is preferred to less. (In regard to item 2, the \$8,000 was a typographical error; it was supposed to be \$800. In regard to item 1, it was supposed to be implied that the other component of the lottery was a 3/4 chance at \$0, but earning credit did not depend on inferring that.)

25. Suppose L_2 is a lottery that Genevieve regards as giving her a 1/4 chance at \$4,000 and a 3/4 chance at 2,000. Suppose also that she prefers \$2,500 to L_2. Is she risk averse, risk neutral, or risk seeking, or do we not have enough information to say for sure?

    She is risk averse, since she prefers a certain sum of money (\$2,500) to a lottery whose expected monetary value is exactly the same as that sum of money: 1/4 × \$4,000 + 3/4 × \$2,000 = \$1,000 + \$1,500 = \$2,500.

questions due Monday, February 21 (section 3.4):

For questions 26–28, assume the following (and notice that these lotteries involve four different amounts of money):

$u(\$90) = u(\$10) + x + y + z$

$u(\$70) = u(\$10) + x + y$

$u(\$40) = u(\$10) + x$

More money is preferred to less.

L_1 is a lottery with a 1/6 chance at \$90 and a 5/6 chance at \$40.

L_2 is a lottery with a 1/2 chance at \$10 and a 1/2 chance at \$70.

L_3 is a lottery with a 1/4 chance at \$90 and a 3/4 chance at \$40.

26. Suppose L_1 is preferred to L_2. What constraint concerning x, y, and z does this imply?

$$
\begin{aligned}
\mathrm{EU}(L_1) &> \mathrm{EU}(L_2) \\
(1/6)u(\$90) + (5/6)u(\$40) &> (1/2)u(\$10) + (1/2)u(\$70) \\
u(\$90) + 5u(\$40) &> 3u(\$10) + 3u(\$70) \\
u(\$10) + x + y + z + 5[u(\$10) + x] &> 3u(\$10) + 3[u(\$10) + x + y] \\
u(\$10) + x + y + z + 5u(\$10) + 5x &> 3u(\$10) + 3u(\$10) + 3x + 3y \\
6x + y + z &> 3x + 3y \\
3x + z &> 2y
\end{aligned}
$$

27. Suppose L_2 is preferred to L_3. What constraint concerning x, y, and z does this imply?

$$
\begin{aligned}
\mathrm{EU}(L_2) &> \mathrm{EU}(L_3) \\
(1/2)u(\$10) + (1/2)u(\$70) &> (1/4)u(\$90) + (3/4)u(\$40) \\
2u(\$10) + 2u(\$70) &> u(\$90) + 3u(\$40) \\
2u(\$10) + 2[u(\$10)\ x + y] &> u(\$10) + x + y + z + 3[u(\$10) + x] \\
2u(\$10) + 2u(\$10) + 2x + 2y &> u(\$10) + x + y + z + 3u(\$10) + 3x \\
2x + 2y &> 4x + y + z \\
y &> 2x + z
\end{aligned}
$$

28. What can be inferred, from your answers to the previous two questions, about the possibility of making the principle of maximizing expected utility agree with the preferences stated in those questions?

    The answer to question 26 implies that $3x + z$ is the upper limit for $2y$. But the answer to question 27 implies that $2x + z$ is the lower limit for y, or (multiplying both sides of the inequality by 2) that $4x + 2x$ is the lower limit for $2y$. But since x and z are both positive numbers, $4x + 2z$ (the lower limit for $2y$) is greater than $2x + z$ (the upper limit for $2y$). This is contradictory, implying that the principle of maximizing expected utility cannot be made to agree with the preferences stated in those questions.

questions due Wednesday, March 2:

For questions 29–30, assume that game G is played by player 1 choosing $x = 1$ or $x = 2$, then player 2 choosing $y = 21$, $y = 22$, $y = 23$, or $y = 24$, then player 1 choosing $z = 1$ or $z = 2$, then payoffs to both players being some function of x, y, and z (which is common knowledge between the players).

29. How many branches does the tree for G have?

    2 options for player 1, then 4 options for player 2, then 2 options for player 1: $2 \times 4 \times 2 = 16$

30. How many rows and columns does the matrix for G have, if player 1 is the row player and player 2 is the column player?

    First, let us ascertain the number of rows by ascertaining the number of strategies for player 1. Any strategy for player 1 will begin with a specification for x, of which there are 2 possibilities. Then the strategy needs to have a plan for what to do in response to any of player 2's specifications for y. At that juncture player 1 has 2 options, and that juncture could come about in any of 4 different ways (based on player 2's having 4 different options for y). So, we have 2^4, or 16. Since this is independent of player 1's 2 different options for his or her specification of x, we have 2×16, or 32.

    Second, we'll ascertain the number of columns by ascertaining the number of strategies for player 2. A strategy for player 2 consists of a plan for what to do in response to any of player 1's specifications for x. At that juncture player 2 has 4 options, and that juncture could come about in any of 2 different ways (based on player 1's having 2 different options for x). So, we have 4^2, or 16.

questions due Friday, March 4:

For questions 31–32, analyze each game using dominance considerations and write the strategy pair(s) corresponding to its solution(s). Write each strategy pair in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively.

31.

	C_1	C_2	C_3	C_4
R_1	9	5	3	5
R_2	4	5	3	7
R_3	8	6	7	9

(R_3, C_2)

32.

	C_1	C_2	C_3	C_4
R_1	5	4	5	6
R_2	4	7	4	9
R_3	2	5	6	8
R_4	1	4	2	8

(R_1, C_1), (R_1, C_2), (R_2, C_1), and (R_2, C_2)

questions due Monday, March 7:

For questions 33–34, state whether the game has any equilibrium strategy pair(s). If it does, write it (or them) in the form (R_x, C_y), where x and y are integers corresponding to row and column numbers, respectively.

33.

	C_1	C_2	C_3
R_1	7	4	9
R_2	9	1	8
R_3	6	4	7

(R_1, C_2) and (R_3, C_2)

34.

	C_1	C_2	C_3	C_4
R_1	6	9	5	7
R_2	6	3	3	4
R_3	7	1	7	5

(None.)

questions due Wednesday, March 9:

For questions 35–36, consider the following game:

	C_1 (q)	C_2 ($1 - q$)
R_1 (p)	9	2
R_2 ($1 - p$)	5	8

35. Suppose $q = 2/3$. Then what is the expected value, for the row player, of playing strategy R_1, and what is the expected value, for the row player, of playing strategy R_2? (Show your work.)

$EU(R_1)$
$= q \times 9 + (1 - q) \times 2$
$= 2/3 \times 9 + (1 - 2/3) \times 2$
$= 18/3 + 1/3 \times 2$
$= 18/3 + 2/3$
$= 20/3$

EU(R_2)
$= q \times 5 + (1 - q) \times 8$
$= 2/3 \times 5 + (1 - 2/3) \times 8$
$= 10/3 + 1/3 \times 8$
$= 10/3 + 8/3$
$= 18/3$

36. Is it possible for the column player's mixed strategy (2/3 C_1, 1/3 C_2) to be one half of a pair of mixed strategies (one mixed strategy for the row player, one mixed strategy for the column player) that is in equilibrium? Why or why not?

No, that is not possible, because if the column player plays that mixed strategy, then the row player's best response will be to play strategy R_1, to which the column player's best response will be to play C_2 rather than any mixed strategy in which C_1 has a non-zero probability of being played—such as (2/3 C_1, 1/3 C_2).

questions due Friday, March 11:

For questions 37–38, consider the following game:

	C_1 (q)	C_2 ($1 - q$)
R_1 (p)	4	13
R_2 ($1 - p$)	9	3

37. Suppose $q = 1/3$. Then what is the expected value, for the row player, of playing the mixed strategy (p R_1, $(1 - p)$ R_2)? (Show your work.) Is it possible for the column player's mixed strategy (1/3 C_1, 2/3 C_2) to be one half of a pair of mixed strategies that is in equilibrium? Why or why not?

EU(p R_1, $(1 - p)$ R_2)
$= p\ [(q)(4) + (1 - q)(13)] + (1 - p)[(q)(9) + (1 - q)(3)]$
$= p\ [(1/3)(4) + (1 - 1/3)(13)] + (1 - p)[(1/3)(9) + (1 - 1/3)(3)]$
$= p\ [(1/3)(4) + (2/3)(13)] + (1 - p)[(1/3)(9) + (2/3)(3)]$
$= p\ (4/3 + 26/3) + (1 - p)(9/3 + 6/3)$
$= p\ (30/3) + (1 - p)(15/3)$
$= (30/3)(p) + 15/3 - (15/3)(p)$
$= (15/3)(p) + 15/3$
$= 5p + 5$

It is not possible for the column player's mixed strategy (1/3 C_1, 2/3 C_2) to be one half of a pair of mixed strategies that is in equilibrium, because setting $q = 1/3$ does not make all of the row player's strategies have the same expected utility, as shown by the fact that EU(p R_1, $(1 - p)$ R_2) is a function of p rather than a constant.

38. Suppose $q = 2/3$. Then what is the expected value, for the row player, of playing the mixed strategy (p R_1, $(1 - p)$ R_2)? (Show your work.) Is it possible for the column player's mixed strategy (2/3 C_1, 1/3 C_2) to be one half of a pair of mixed strategies that is in equilibrium? Why or why not?

EU(p R_1, $(1 - p)$ R_2)
$= p\ [(q)(4) + (1 - q)(13)] + (1 - p)[(q)(9) + (1 - q)(3)]$
$= p\ [(2/3)(4) + (1 - 2/3)(13)] + (1 - p)[(2/3)(9) + (1 - 2/3)(3)]$
$= p\ [(2/3)(4) + (1/3)(13)] + (1 - p)[(2/3)(9) + (1/3)(3)]$
$= p\ (8/3 + 13/3) + (1 - p)(18/3 + 3/3)$
$= p\ (21/3) + (1 - p)(21/3)$
$= (21/3)(p) + 21/3 - (21/3)(p)$
$= 21/3$
$= 7$

It is possible for the column player's mixed strategy (2/3 C_1, 1/3 C_2) to be one half of a pair of mixed strategies that is in equilibrium, because setting $q = 2/3$ makes all of the row player's strategies have the same expected utility, as shown by the fact that EU(p R_1, $(1 - p)$ R_2) is a constant rather than a function of p.

questions due Wednesday, March 16:

For questions 39–41, consider the following game:

	C_1 (q)	C_2 $(1 - q)$
R_1 (p)	16	4
R_2 $(1 - p)$	6	9

39. Suppose that the expected utility of the foregoing game for the column player is a formula involving q (where q is her probability of playing C_1 as opposed to C_2) in which the coefficient of q ends up being 0, because of the strategy (p R_1, $(1 - p)$ R_2) that the row player happens to be playing. Derive the value of p that makes this the case, by starting with (1) a formula expressing the expected utility of the game for the row player, (2) a formula expressing the expected utility of the game for the column player, or (3) a formula expressing the value of the game, and then—after you have started with one of those three formulas—simplifying that formula until you can isolate an expression involving p that you can then use to solve for p. (Show your work, of course.)

EU(q C_1, $(1 - q)$ C_2)
$= q\ [(p)(16) + (1 - p)(6)] + (1 - q)[(p)(4) + (1 - p)(9)]$
$= q\ (16p + 6 - 6p) + (1 - q)(4p + 9 - 9p)$
$= q\ (10p + 6) + (1 - q)(-5p + 9)$
$= q\ (10p + 6) + (-5p + 9) - (q)(-5p + 9)$
$= q\ (10p + 6 + 5p - 9) + (-5p + 9)$
$= q\ (15p - 3) + (-5p + 9)$

$15p - 3 = 0$
$15p = 3$
$p = 3/15$
$p = 1/5$

40. Follow the same instructions as for question 39, except from the other player's point of view. So, in those instructions, reverse the words 'row' and 'column', the letters 'R' and 'C', the letters 'p' and 'q', etc. Since you derived a particular value of p there, here you will derive a particular value of q.

EU(p R_1, $(1 - p)$ R_2)
$= p\ [(q)(16) + (1 - q)(4)] + (1 - p)[(q)(6) + (1 - q)(9)]$
$= p\ (16q + 4 - 4q) + (1 - p)(6q + 9 - 9q)$
$= p\ (12q + 4) + (1 - p)(-3q + 9)$
$= p\ (12q + 4) + (-3q + 9) - (p)(-3q + 9)$
$= p\ (12q + 4 + 3q - 9) + (-3q + 9)$
$= p\ (15q - 5) + (-3q + 9)$

$15q - 5 = 0$
$15q = 5$
$q = 5/15$
$q = 1/3$

41. Using the values of p and q that you derived, compute the value of the game.

EU(game)
= pr(R_1)pr(C_1)(R_1, C_1) + pr(R_1)pr(C_2)(R_1, C_2) + pr(R_2)pr(C_1)(R_2, C_1) + pr(R_2)pr(C_2)(R_2, C_2)
= pq(R_1, C_1) + p(1 – q)(R_1, C_2) + (1 – p)(q)(R_2, C_1) + (1 – p)(1 – q)(R_2, C_2)
= (1/5)(1/3)(16) + (1/5)(1 – 1/3)(4) + (1 – 1/5)(1/3)(6) + (1 – 1/5)(1 – 1/3)(9)
= (1/5)(1/3)(16) + (1/5)(2/3)(4) + (4/5)(1/3)(6) + (4/5)(2/3)(9)
= 16/15 + 8/15 + 24/15 + 72/15
= 120/15
= 8

questions due Friday, March 18:

For questions 42–44, consider the following game:

	C_1 (q)	C_2 ($1 - q$)
R_1 (p)	8	15
R_2 ($1 - p$)	11	5

42. Suppose that the expected utility of the foregoing game for the column player is $q(-3p + 2) + (5p - 1)$. What value of p makes the column player indifferent among all possible values of q (assuming that her only interest in the value of q is based on her interest in influencing the expected utility of the game)?

$-3p + 2 = 0$
$-3p = -2$
$p = 2/3$

43. Suppose that the expected utility of the foregoing game for the row player is $p(9q - 7) + (2q + 4)$. What value of q makes the row player indifferent among all possible values of p (assuming that his only interest in the value of p is based on his interest in influencing the expected utility of the game)?

$9q - 7 = 0$
$9q = 7$
$q = 7/9$

44. What values of p and q make (p R_1, (1 – p) R_2; q C_1, (1 – q) C_2) an equilibrium strategy pair? (Note: Do not assume that the expressions given in questions 42 and 43 actually have anything to do with the game.)

$p = 6/13$, $q = 10/13$

questions due Monday, March 28:

For questions 45–47, consider the following game:

	C_1 (q)	C_2 ($1 - q$)
R_1 (p)	12	7
R_2 ($1 - p$)	4	9

45. Consider the strategy pair (1/2 R_1, 1/2 R_2; 1/5 C_1, 4/5 C_2). Verify that the column player's mixed strategy (1/5 C_1, 4/5 C_2) is eligible to be one half of an equilibrium strategy pair.

$EU(p\ R_1, (1 - p)\ R_2)$
$= p \times EU(R_1) + (1 - p) \times EU(R_2)$
$= p\ [(q)(12) + (1 - q)(7)] + (1 - p)[(q)(4) + (1 - q)(9)]$
$= p\ (12q + 7 - 7q) + (1 - p)(4q + 9 - 9q)$
$= p\ (5q + 7) + (1 - p)(-5q + 9)$
$= p\ (5q + 7) + (-5q + 9) - (p)(-5q + 9)$
$= p\ (5q + 7) + (-5q + 9) + (p)(5q - 9)$
$= p\ (10q - 2) + (-5q + 9)$

$10q - 2 = 0$
$10q = 2$
$q = 2/10$
$q = 1/5$

Or q can be computed as follows: $q = (9 - 7)/[(9 - 7) + (12 - 4)] = 2/(2 + 8) = 2/10 = 1/5$.

46. Suppose that the column player thinks that the mixed strategy (1/2 C_1, 1/2 C_2) is what she should play in order to play her half of an equilibrium strategy pair. Show that this belief would be mistaken.

$EU(p\ R_1, (1 - p)\ R_2)$
$= p \times EU(R_1) + (1 - p) \times EU(R_2)$
$= p\ [(q)(12) + (1 - q)(7)] + (1 - p)[(q)(4) + (1 - q)(9)]$
$= p\ [(1/2)(12) + (1 - 1/2)(7)] + (1 - p)[(1/2)(4) + (1 - 1/2)(9)]$
$= p\ [(1/2)(12) + (1/2)(7)] + (1 - p)[(1/2)(4) + (1/2)(9)]$
$= p\ (12/2 + 7/2) + (1 - p)(4/2 + 9/2)$
$= p\ (19/2) + (1 - p)(13/2)$
$= 19p/2 + 13/2 - 13p/2$
$= 6p/2 + 13/2$
$= 3p + 13/2$

Since $q = 1/2$ does not make the row player's expected utility independent of p, it does not determine a mixed strategy for the column player that can be one half of an equilibrium strategy pair.

47. Suppose that the column player is playing the mixed strategy (1/2 C_1, 1/2 C_2). Analyze the game from the row player's point of view and explain what strategy (whether pure or mixed) would maximize his expected utility.

Based on the calculations done for question 46, the row player's expected utility is $3p + 13/2$. Since that is positively influenced by the value of p, it is maximized when p is maximized, which is when $p = 1$. So, the row player should play the mixed strategy (1/2 R_1, 1/2 R_2), which is just the pure strategy R_1.

questions due Wednesday, March 30:

48. Is the following game a coordination game? Why or why not?

	C_1	C_2
R_1	2, 3	5, 6
R_2	8, 7	3, 4

No, because there is a single outcome—the one resulting from (R_2, C_1)—that is best for both players.

49. Is the following game a coordination game? Why or why not?

	C_1	C_2
R_1	6, 19	1, 13
R_2	4, 16	8, 17

Yes, because there are two equilibrium outcomes favored by both players—the ones resulting from (R_1, C_1) and (R_2, C_2)—but the players rank them differently.

50. Is the following game a coordination game? Why or why not?

	C_1	C_2
R_1	6, 8	9, 7
R_2	8, 2	4, 3

No, because there is no equilibrium outcome.

questions due Friday, April 1:

51. Is the following game a prisoner's dilemma? Why or why not?

	C_1	C_2
R_1	3, 4	8, 3
R_2	4, 5	6, 8

No, because neither player has a dominant strategy, and because there is no equilibrium outcome.

52. Is the following game a prisoner's dilemma? Why or why not?

	C_1	C_2
R_1	7, 6	–3, –1
R_2	–4, 0	5, 8

No, because neither player has a dominant strategy, and because there are two equilibrium outcomes.

53. Is the following game a prisoner's dilemma? Why or why not?

	C_1	C_2
R_1	–4, 20	4, 10
R_2	–2, 4	40, 2

Yes, because each player has a dominant strategy, and the resulting strategy pair leads to an equilibrium outcome that is Pareto-inferior to another outcome.

questions due Monday, April 4:

54. Is (or are) any of the following outcomes Pareto optimal? If so, which one(s)?

	C_1	C_2
R_1	5, 3	8, 2
R_2	6, 1	5, 7

Yes, the second and fourth ones are. The first one is not Pareto optimal since the last one is better (than the first one) for the column player and just as good for the row player. The second one (8, 2) is Pareto optimal since the row player, enjoying his maximum (8) there, would not consent to move away to any other outcome. The third one (6, 1) is not Pareto optimal since the second one is better (than the third one) for both players. The fourth one (5, 7) is Pareto optimal since the column player, enjoying her maximum (7) there, would not consent to move away to any other outcome.

55. Consider applying the model of p. 222 to the following prisoner's dilemma. What should we take the values of *u* and *u'* to be, in order to apply that model?

	C_1	C_2
R_1	2/3, 2/3	0, 1
R_2	1, 0	1/2, 1/2

$u = 1/2$, $u' = 2/3$

56. Apply the model of p. 222 to the following prisoner's dilemma. If $p = 1/2$ and $q = 1/3$, what must be true of *r* in order for being a cooperator to have a higher expected utility than being a cheater?

	C_1	C_2
R_1	3/4, 3/4	0, 1
R_2	1, 0	1/4, 1/4

EU(cooperator) > EU(cheater)
$u + pr(u' - u) - q(1 - p)u > u + pq(1 - u)$
$pr(u' - u) - q(1 - p)u > pq(1 - u)$
$(1/2)r(3/4 - 1/4) - (1/3)(1 - 1/2)(1/4) > (1/2)(1/3)(1 - 1/4)$
$(1/2)r(1/2) - (1/3)(1/2)(1/4) > (1/2)(1/3)(3/4)$
$(1/4)r - 1/24 > 3/24$
$(1/4)r > 4/24$
$r > 16/24$
$r > 2/3$

questions due Wednesday, April 13 (pp. 261–273):

57. How many profiles can be constructed from a situation in which there are four alternatives and four people?

31 million (or, more specifically, 75^4, or 31,640,625)

58. Suppose you are presented with the following profile. How many other profiles are there that have the same number of alternatives and the same number of people?

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>
a, *b*	*c*	*a*, *b*, *c*	*c*
c	*b*		*a*, *b*
	a		

28,560

59. Same question as for no. 58, but in regard to the following profile:

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>	<u>G</u>	<u>H</u>	<u>I</u>
a	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*

0

questions due Friday, April 15 (pp. 275–285):

For questions 60–62, let F be a social welfare function, and consider the following profiles and F-determined corresponding social preference orderings:

1:

A	B	C	soc.
a	*d*	*d*	*a*
b	*c*	*c*	*b*
d	*a*	*b*	*c*
c	*b*	*a*	*d*

2:

A	B	C	soc.
a	*d*	*d*	*b*
d	*c*	*c*	*d*
b	*a*	*a*	*a*
c	*b*	*b*	*c*

60. If we look only at the positions of alternatives *a* and *b*, do these profiles and corresponding social preference orderings prove that F violates condition I? How about if we look only at the positions of alternatives *c* and *d*?

    No; Yes.

61. If we look only at the positions of alternatives *a* and *c*, do these profiles and corresponding social preference orderings prove that F violates condition I? How about if we look only at the positions of alternatives *b* and *d*?

    No; No.

62. Are there any other pairs of alternatives that one would need to check (in order to continue to test F for compliance with condition I), if the answers to questions 60 and 61 were all negative? If so, what are they?

    Yes—*a* & *d*, and *b* & *c*.

questions due Monday, April 18 (pp. 287–294):

63. For the following profile, figure out the social preference ordering determined by pairwise majority rule. If there is not one, show why.

A	B	C	D
c	*a, d*	*b*	*d*
b	*b, c*	*a, c*	*b*
a		*d*	*c*
d			*a*

The social preferences determined by pairwise majority rule can be ascertained via the following table:

pair	number preferring first alternative	number preferring second alternative	social preference
a vs. *b*	1	3	*b* P *a*
a vs. *c*	1	2	*c* P *a*
a vs. *d*	2	1	*a* P *d*
b vs. *c*	2	1	*b* P *c*
b vs. *d*	2	2	*b* I *d*
c vs. *d*	2	2	*c* I *d*

These social preferences fail to determine a social preference ordering because there are cyclical. In fact, they are cyclical in several different ways. Here are some examples: {*a* P *d*, *d* I *b*, *b* P *a*}, {*a* P *d*, *d* I *c*, *c* P *a*}, {*b* P *c*, *c* I *d*, *d* I *b*}.

64. Same instructions as for no. 63, but in regard to the following profile:

A	B	C	D
c, d	*d*	*b*	*a, c, d*
b	*a*	*c*	*b*
a	*b, c*	*d*	
		a	

The social preferences determined by pairwise majority rule can be ascertained via the following table:

pair	number preferring first alternative	number preferring second alternative	social preference
a vs. *b*	2	2	*a* I *b*
a vs. *c*	1	2	*c* P *a*
a vs. *d*	0	3	*d* P *a*
b vs. *c*	1	2	*c* P *b*
b vs. *d*	1	3	*d* P *b*
c vs. *d*	1	1	*c* I *d*

These social preferences, in turn, determine the following social preference ordering:

c, *d*
a, *b*

65. What can be concluded, strictly from question 63 and your answer to it, about whether pairwise majority rule satisfies condition U? How about from question 64 and your answer to it?

From question 63 and its answer, it can be concluded that pairwise majority rule does *not* satisfy condition U. From question 64 and its answer, no conclusion can be drawn about whether pairwise majority rule satisfies condition U.

questions due Wednesday, April 20 (pp. 295–312):

The following profile is for questions 66–67.

A	B	C	D	E
c	*b*	*a*	*c*	*a*
d	*c*	*c*	*d*	*c*
b	*d*	*b*	*b*	*d*
a	*a*	*d*	*a*	*b*

66. What social preference ordering would be dictated by plurality voting?

We have the following tally of first-place votes:

alternative	number of first-place votes
a	2
b	1
c	2
d	0

This yields the following social preference ordering:

a, *c*
b
d

67. What social preference ordering would be dictated by instant runoff voting?

Alternative *d* is eliminated first, then *b*, then *a*, yielding the following social preference ordering:

c
a
b
d

For questions 68–69, let F be a social welfare function, and consider the following profiles and F-determined corresponding social preference orderings:

1:

A	B	C	soc.
c	*b*	*a*	*b*
a	*d*	*c*	*a*
d	*c*	*b*	*c*
b	*b*	*d*	*d*

2:

A	B	C	soc.
c	*b*	*a*	*b*
a	*c*	*c*	*a*
d	*d*	*b*	*d*
b	*a*	*d*	*c*

3:

A	B	C	soc.
b	*b*	*d*	*b*
a	*c*	*c*	*c*
c	*d*	*a*	*a*
d	*a*	*b*	*d*

4:

A	B	C	soc.
a	*b*	*d*	*c*
b	*c*	*c*	*b*
c	*d*	*a*	*a*
d	*a*	*b*	*d*

68. Do profiles 1 and 2 and their corresponding social preference orderings prove that F violates condition M?

Due to an error in the preference ordering for person B in profile 1 (alternative *b* listed twice), this is not a valid question. But if the second '*b*' is changed to '*a*' (so that the preference ordering more closely resembles the preference ordering of person B in profile 2), then the profiles and their corresponding preference orderings prove that F violates condition M, because alternative *c* moves up in the preference ordering of person B (going from profile 1 to profile 2) but moves down in the social preference ordering.

69. Do profiles 3 and 4 and their corresponding social preference orderings prove that F violates condition M?

No.

questions due Friday, April 22 (pp. 313–324):

70. Consider the following three profiles (each concerning two alternatives and one person) and corresponding social preference orderings, supposedly produced by social welfare function F.

Profile 1: A	s.p.o.:	Profile 2: A	s.p.o.:	Profile 3: A	s.p.o.:
a	*a*	*b*	*b*	*a, b*	*a*
b	*b*	*a*	*a*		*b*

Which of the following is true?

a. These profiles and social preference orderings imply that F satisfies condition NI.
b. These profiles and social preference orderings imply that F violates condition NI.
c. These profiles and social preference orderings do not imply either that F satisfies condition NI or that F violates condition NI.

b

71. Consider the following profile. What are the social preference orderings that could be generated by a social welfare function that violates condition ND?

A	B	C
b	*a*	*d*
d	*b*	*c*
c	*d*	*b*
a	*c*	*a*

The possibilities are the preference orderings held by the individual people—that is, any of these three:

b	*a*	*d*
d	*b*	*c*
c	*d*	*b*
a	*c*	*a*

72. Consider the following profile. What preferences does condition P require a social welfare function to include in whatever social preference ordering it determines for this profile?

A	B	C	D
a	*c*	*d*	*a*
d	*a*	*a*	*c*
c	*d*	*b*	*d*
b	*b*	*c*	*b*

a P *b* and *d* P *b*

questions due Wednesday, April 27 (pp. 325–333):

73. What are the two different distinctions that can be made in order to classify the conditions on social welfare functions?

    One is the distinction between conditions that are democratic and conditions that are merely structural. The other is the distinction between conditions that are concerned with the existence of certain social preference orderings and conditions that are concerned with certain kinds of consistency between social preference orderings.

74. Is it possible for two profiles and their corresponding F-determined social preference orderings to prove that F violates condition I but not to prove that F violates condition M? If so, give an example of two such profiles and their corresponding social preference orderings.

    Yes, this is possible. Here is an example illustrating this possibility:

Profile 1: A	Profile 1: B	s.p.o.:	Profile 2: A	Profile 2: B	s.p.o.:
a	*a*	*a*	*a*	*a*	*c*
b	*b*	*b*	*b*	*c*	*a*
c	*c*	*c*	*c*	*b*	*b*

75. Is it possible for two profiles and their corresponding F-determined social preference orderings to prove that F violates both condition P and condition ND? If so, give an example of two such profiles and their corresponding social preference orderings.

    No, this is not possible.

questions due Friday, April 29 (pp. 335–338):

76. Suppose you were working on a proof in which one of the statements would concern how a particular social welfare function F generates a social preference ordering for a profile with certain stipulated characteristics—for example, a profile in which twice as many people prefer *g* to *h* as prefer *h* to *g*. What condition would you need to assume that F satisfies, in order for such a statement to be a justified part of your proof?

    Condition U.

77. Suppose you were working on a proof in which one of the statements would concern the profile for which a social welfare function generates, as a social preference ordering, a preference ordering with certain stipulated characteristics—for example, a preference ordering in which *g* is ranked above *h* but below *i*. What condition would you need to assume that F satisfies, in order to include such a statement in your proof?

    Condition NI.

78. Let Q be the proposition that social welfare function F determines the following social preference ordering for the following profile:

profile:

A	B	C	s.p.o.
d	*c*	*d*	*c*
a	*d*	*c*	*b*
b	*b*	*a*	*a*
c	*a*	*b*	*d*

What is a statement that you can infer from proposition Q, if you also assume that F satisfies condition I?

You can infer any of at least six statements, each corresponding to one of the six pairs into which the alternatives can be put (*a* and *b*, *a* and *c*, *a* and *d*, *b* and *c*, etc.) Here is the statement concerning *a* and *b* that you can infer: In any profile in which A prefers *a* to *b*, B prefers *b* to *a*, and C prefers *a* to *b*, the social preference ordering determined by F will rank *b* above *a*.

questions due Monday, May 2 (pp. 339–344):

79. What is an example of a theorem that is stronger than Arrow's impossibility theorem? That is, what is an example of theorem T such that T implies Arrow's impossibility theorem, but Arrow's impossibility theorem does not imply T?

Any statement like Arrow's impossibility theorem but mentioning only a subset of the conditions mentioned by that theorem is stronger than that theorem. Also, any statement like Arrow's impossibility theorem but not having any restrictions concerning the number of alternatives or the number of people is stronger than that theorem. So, here is a good answer: No social welfare function satisfies conditions U, I, and M. (Of course, this statement is not true, but that is o.k.—it does not have to be true in order to be stronger than Arrow's impossibility theorem.)

80. In proving Arrow's impossibility condition, what are the four conditions on which the proof focuses, instead of the five conditions mentioned in the theorem (U, I, M, NI, and ND)?

Conditions U, I, ND, and P.

81. Suppose set *S* is quasi-decisive for alternative *k* over alternative *r*. What else do we need to know in order to legitimately infer that *k* is ranked above *r* in the social preference ordering?

We need to know that everyone in *S* ranks *k* above *r* and that everyone not in *S* ranks *r* above *k*.